A 3" × 1.2" brass ($\sigma = 2.5 \times 10^7$ S/m) rectangular waveguide is filled with ABS plastic ($\varepsilon_r = 2$, $\tan\delta = 0.005$). In order, find the cutoff frequency of the: a) lowest four TE modes and b) lowest two TM modes. Neglect loss tangent.

Per (3.84), $f_{c,mn} = \dfrac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\dfrac{m\pi}{a}\right)^2 + \left(\dfrac{n\pi}{b}\right)^2}$ where $m = 0, 1, 2, \ldots$, $n = 0, 1, 2, \ldots$, and $m = n \neq 0$ for TE modes and $m = 1, 2, \ldots$ and $n = 1, 2, \ldots$ for TM modes.

Dimensions: $a = 3"(0.0254\text{ m/in}) = 0.0762$ m and $b = 1.2"(0.0254\text{ m/in}) = 0.03048$ m

a) TE mode cutoff frequencies

$$\text{TE}_{10},\ f_{c,10} = \frac{1}{2\pi\sqrt{4\pi\times10^{-7}(2)8.85419\times10^{-12}}}\sqrt{\left(\frac{\pi}{0.0762}\right)^2} = 3.37385\times10^7\left(\frac{\pi}{0.0762}\right)$$

$$\Rightarrow \underline{\boldsymbol{f_{c,10} = 1.39098 \times 10^9\text{ Hz} = 1.391\text{ GHz}}}.$$

$$\text{TE}_{20},\ f_{c,20} = 3.37385\times10^7\sqrt{\left(\frac{2\pi}{0.0762}\right)^2} \quad \Rightarrow \underline{\boldsymbol{f_{c,20} = 2.78196 \times 10^9\text{ Hz} = 2.782\text{ GHz}}}.$$

$$\text{TE}_{01},\ f_{c,01} = 3.37385\times10^7\sqrt{\left(\frac{\pi}{0.03048}\right)^2} \quad \Rightarrow \underline{\boldsymbol{f_{c,01} = 3.47745 \times 10^9\text{ Hz} = 3.477\text{ GHz}}}.$$

$$\text{TE}_{11},\ f_{c,11} = 3.37385\times10^7\sqrt{\left(\frac{\pi}{0.0762}\right)^2 + \left(\frac{\pi}{0.03048}\right)^2}$$

$$\Rightarrow \underline{\boldsymbol{f_{c,11} = 3.7453 \times 10^9\text{ Hz} = 3.745\text{ GHz}}}.$$

b) TM mode cutoff frequencies

$$\text{TM}_{11},\ f_{c,11} = 3.37385\times10^7\sqrt{\left(\frac{\pi}{0.0762}\right)^2 + \left(\frac{\pi}{0.03048}\right)^2}$$

$$\Rightarrow \underline{\boldsymbol{f_{c,11} = 3.7453 \times 10^9\text{ Hz} = 3.745\text{ GHz}}}.$$

$$\text{TM}_{21},\ f_{c,21} = 3.37385\times10^7\sqrt{\left(\frac{2\pi}{0.0762}\right)^2 + \left(\frac{\pi}{0.03048}\right)^2}$$

$$\Rightarrow \underline{\boldsymbol{f_{c,21} = 4.4533 \times 10^9\text{ Hz} = 4.453\text{ GHz}}}.$$