3.5 A 10 cm length of a K-band copper waveguide is filled with a dielectric material with $\epsilon_r = 2.55$ and $\tan\delta = 0.0015$. If the operating frequency is 15 GHz, find the total loss through the guide and the phase delay from the input to the output of the guide.

From Appendix I, K-band waveguide WR-42 has dimensions 0.420" x 0.170" or 1.0668 cm x 0.4318 cm (a b)

Per (3.85), $f_{c10} = \frac{1}{2a\sqrt{\mu\epsilon}} = \frac{1}{2(0.010668)\sqrt{4\pi\times10^{-7}(2.55)8.8541878\times10^{-12}}}$

$$f_{c10} = 8.799087\ \text{GHz}$$

Check next highest mode (f_{c20}) cut-off frequency

Per (3.84), $f_{c20} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{2\pi}{0.010668}\right)^2 + 0} = 2f_{c10}$

$$= 17.598\ \text{GHz}$$

$\Rightarrow$ At 15 GHz, we <u>only</u> have TE_{10} mode

Per (3.7), $k = \omega\sqrt{\mu\epsilon} = \omega/v_p = \frac{\omega}{c/\sqrt{\epsilon_r}}$

$$= \frac{2\pi(15\times10^9)\sqrt{2.55}}{2.99792458\times10^8} = 502.01942\ \frac{\text{rad}}{\text{m}}$$

Per (3.91), $\beta_{10} = \sqrt{k^2 - (\pi/a)^2} = \sqrt{502.02^2 - (\pi/0.010668)^2}$

$$= 406.57177\ \text{rad/m}$$

Phase delay $= \beta_{10}\ell = 406.57(0.10) = 40.657\ \text{rad}$

$$= 2329.5^\circ$$

Per (3.29), $\alpha_d = \frac{K^2 \tan\delta}{2\beta} = \frac{502.02^2(0.0015)}{2(406.572)}$

$= 0.464906 \text{ Np/m}$

Per (3.96), $\alpha_c = \frac{R_s}{a^3 b \beta K \eta}\left(2b\pi^2 + a^3 K^2\right)$

where $R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{2\pi(15\times10^9)4\pi\times10^{-7}}{2(5.813\times10^7)}} = 0.031917\ \Omega$

$\eta = \sqrt{\mu/\varepsilon} = \sqrt{\frac{4\pi\times10^{-7}}{2.55(8.8541878\times10^{-12})}} = 235.9177\ \Omega$

$\alpha_c = \frac{0.032\left[2(0.004318)\pi^2 + 0.010668^3(502.02^2)\right]}{0.010668^3(0.004318)406.572(502.02)235.92}$

$= 0.0494638 \text{ Np/m}$

$\alpha = \alpha_c + \alpha_d = 0.464906 + 0.049464 = 0.51437 \text{ Np/m}$

$= 4.46776 \text{ dB/m}$

Total loss $= \alpha\ell = 4.46776 \text{ dB/m}\,(0.1\text{ m})$

Total loss $= 0.4468$ dB