A lossless transmission line (50 Ω, 2.4×10^8 m/s) of length 15 cm has a measured input impedance of $18 + j32\ \Omega$ at 2 GHz. Using a Smith chart, find: a) input reflection coefficient, b) input admittance, c) load reflection coefficient, d) SWR, e) return loss, and f) load impedance.

Calculate wavelength to be $\lambda = v_p / f = 2.4 \times 10^8 / 2 \times 10^9 \Rightarrow \lambda = 0.12$ m = 12 cm.

Normalize and plot input impedance

- $z_{in} = Z_{in} / Z_0 = (18 + j\,32) / 50 \Rightarrow z_{in} = 0.36 + j\,0.64\ \Omega/\Omega$.
- Plot z_{in} on Smith chart by finding intersection of $r = 0.36$ circle with $x = 0.64$ arc.

a) Find input reflection coefficient

- Set compass to distance between center of Smith chart and z_{in}. Use compass to mark the "RFL COEFF, E or I" scale below Smith chart on left side to determine $|\Gamma_{in}| = 0.6$.
- Use a straight edge to draw radial line from center of Smith chart through z_{in} and outer rings of Smith chart. Use the "ANGLE OF REFLECTION COEFFICIENT IN DEGREES" scale to read $\angle\,\Gamma_{in} = 110°$.
- Put magnitude and angle together to get $\Rightarrow$ **$\Gamma_{in} = 0.6\angle 110°$**.

b) Find input admittance

- Set compass to distance between center of Smith chart and z_{in}. Use compass and draw circle centered on Smith chart.
- Use a straight edge to extend radial line from center of Smith chart through z_{in} to other side of Smith chart.
- Read normalized input admittance at intersection of circle and radial line at 180° opposite to z_{in} as $y_{in} = 0.67 - j\,1.19$ S/S.
- Find input admittance by dividing y_{in} by Z_0. $Y_{in} = y_{in} / Z_0 = (0.67 - j\,1.19\ \text{S/S}) / 50\ \Omega$

  $\Rightarrow$ **$Y_{in} = 13.4 - j\,23.8$ mS**.

c) Find load reflection coefficient

- Calculate $\ell/\lambda = 15 / 12 = 1.25$. Subtract $2(0.5) = 1$ (i.e., remove integer multiples of $n\lambda/2$) to get $\Rightarrow \ell/\lambda = 0.25$. A complete circle on a Smith Chart is 0.5λ So, Γ_L is 180° around the circle of constant $|\Gamma| = 0.6$ from the z_{in} & Γ_{in} point, i.e., at same location as y_{in}!
- Use the "ANGLE OF REFLECTION COEFFCIENT IN DEGREES" scale to read $\angle\,\Gamma_L = -70°$ and note $|\Gamma_L| = |\Gamma_{in}| = 0.6$.
- Put magnitude and angle together to get $\Rightarrow$ **$\Gamma_L = 0.6\angle -70°$**.

d) Find SWR

- Use compass to draw $|\Gamma| = 0.6$ arc, centered on Smith chart scales, through 'SWR' scale below Smith chart on left side. Read $\Rightarrow$ **SWR = 4** .

e) Find return loss

- Use compass to draw $|\Gamma| = 0.6$ arc, centered on Smith chart scales, through 'RTN LOSS (dB)' scale below Smith chart on left side. Read $\Rightarrow$ **RL = 4.4 dB** .

f) Find load impedance

- At Γ_L (also y_{in}) point, read-off the normalized load resistance $r_L = 0.67$ and load reactance $x_L = -1.19$, giving $z_L = 0.67 - j\ 1.19\ \Omega/\Omega$.
- Find load impedance by multiplying z_L with Z_0 to get $Z_L = Z_0 z_L = 50(0.67 - j\ 1.19)$ $\Rightarrow$ $\mathbf{Z_L = 33.5 - j\ 59.5\ \Omega}$.

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Bonus:

For comparison, the exact values are-

$\mathbf{\Gamma_{in} = 0.6022\angle 109.8°}$

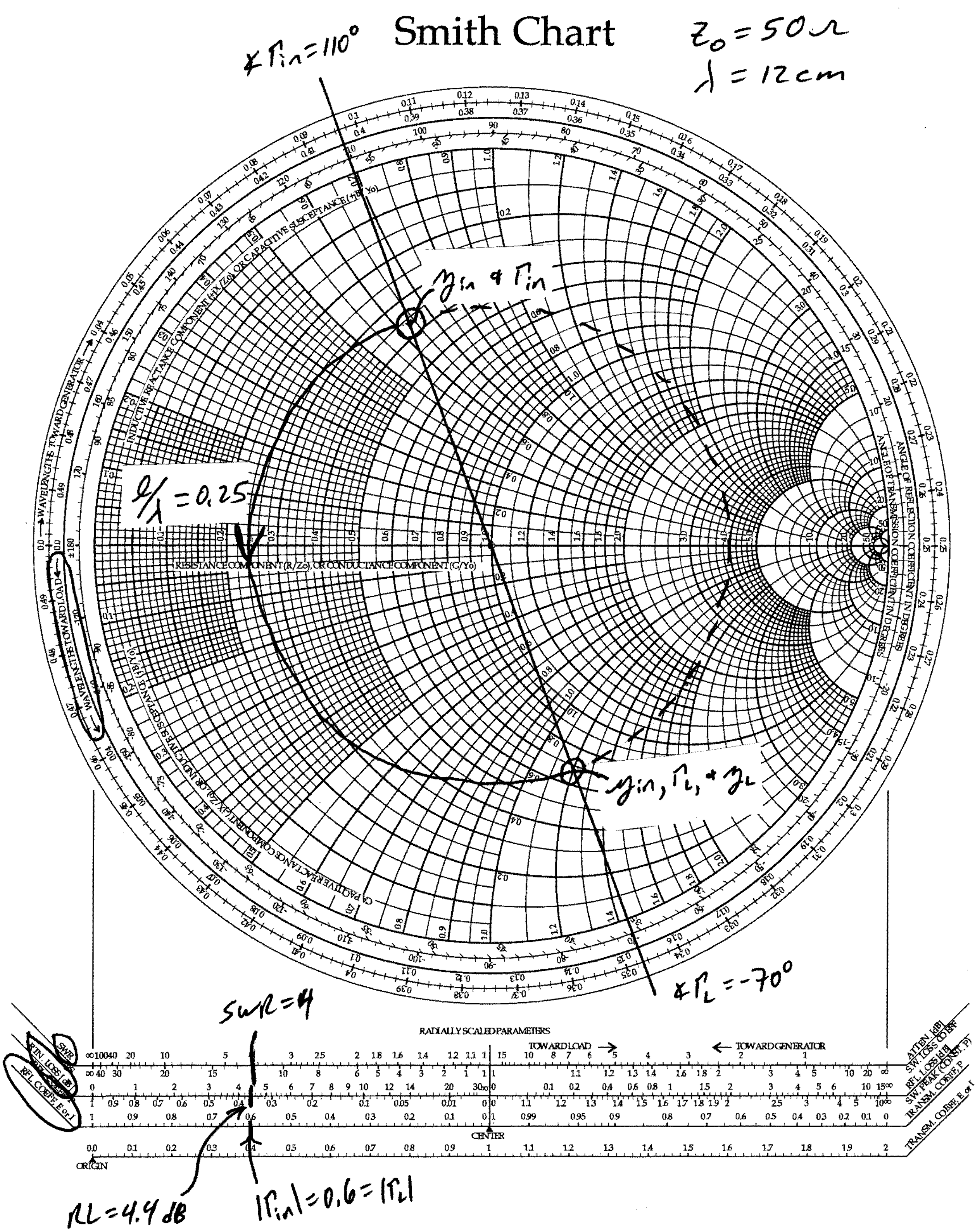
$\mathbf{Y_{in} = 13.35 - j\ 23.74\ mS}$

$\mathbf{\Gamma_L = 0.6022\angle -70.2°}$

SWR = 4.027

RL = 4.406 dB

$\mathbf{Z_L = 33.38 - j\ 59.35\ \Omega}$


Smith Chart
$Z_0 = 50\Omega$
$\lambda = 12cm$
$\angle\Gamma_{in} = 110^\circ$
z_{in} & Γ_{in}
$\ell/\lambda = 0.25$
$y_{in}, \Gamma_L, + z_L$
$\angle\Gamma_L = -70^\circ$
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
SWR = 4
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
SWR
RTN. LOSS [dB]
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, E or I
TRANSM. COEFF, P
CENTER
ORIGIN
RL = 4.4 dB
$|\Gamma_{in}| = 0.6 = |\Gamma_L|$