Using a lossless transmission line (75 Ω and $v_p = 2.3\times10^8$ m/s), find the shortest a) open and b) short circuit stubs that appear as a 1 pF capacitor at 1.6 GHz.

Find phase constant (2.11): $\beta = \dfrac{\omega}{v_p} = \dfrac{2\pi(1.6\times10^9)}{2.3\times10^8} \Rightarrow \beta = 43.709115$ rad/m.

Find 1 pF capacitor impedance:

$$Z_{1\text{pF}} = \frac{1}{j\omega C} = \frac{1}{j2\pi(1.6\times10^9)1\times10^{-12}} \Rightarrow Z_{1\text{pF}} = -j99.47184\ \Omega.$$

a) Determine shortest possible length for an open circuit stub:

$$Z_{OC} = Z_{2\text{nH}}$$

$$\frac{Z_0}{j\tan(\beta\ell_{oc})} = \frac{75}{j\tan(43.709115\ell_{oc})} = -j99.47184\ \Omega$$

$$\ell_{oc} = \frac{\tan^{-1}(75/99.47184)}{43.709115}$$

$$\Rightarrow \quad \underline{\ell_{oc} = 0.01478055\text{ m} = 1.4781\text{ cm}}.$$

b) Determine shortest possible length for a short circuit stub:

$$Z_{SC} = Z_{2\text{nH}}$$

$$jZ_0\tan(\beta\ell_{sc}) = j75\tan(43.709\ell_{sc}) = -j99.472\ \Omega$$

$$\ell_{sc} = \frac{\tan^{-1}(-99.472/75)}{43.709}$$

$$\Rightarrow \quad \underline{\ell_{sc} = -0.02115695\text{ m (not realizable)}}$$

Remember that one can always add integer multiples of half wavelength to transmission line lengths and get same impedance. Here the wavelength is:

$$\lambda = \frac{u}{f} = \frac{2.3\times10^8}{1.6\times10^9} = 0.14375\text{ m} = 14.375\text{ cm} \Rightarrow \underline{\lambda/2 = 7.1875\text{ cm}}.$$

So, the shortest possible realizable short circuit stub would have length:

$$\ell_{sc} = -2.115695 + 7.1875 \Rightarrow \underline{\ell_{sc} = 5.0718\text{ cm}}.$$

Open circuit stub wins! However, both are equivalent to a 1 pF capacitor at 1.6 GHz.