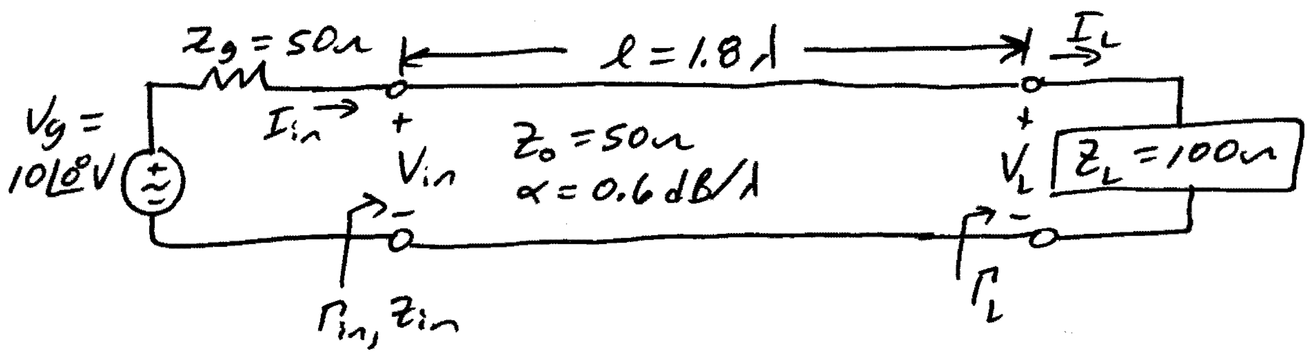
2.29 A 50 Ω transmission line is matched to a 10 V source and feeds a load $Z_L = 100\ \Omega$. If the line is **1.8**λ long and has an attenuation constant $\alpha = 0.6$ dB/λ, find the powers that are delivered by the source, lost in the line, and delivered to the load.

- Find both P_{in} and P_{Vg}. Analytic solution.



$$\alpha = 0.6\ \text{dB}/\lambda \left(\frac{1\ \text{Np}}{20\log e\ \text{dB}}\right) = 0.069078\ \text{Np}/\lambda$$

$$(2.10)\quad \lambda = 2\pi/\beta \Rightarrow \beta = 2\pi/\lambda$$

$$(2.5)\quad \gamma = \alpha + j\beta = (0.069078 + j\,2\pi)/\lambda$$

$$(2.35)\quad \Gamma = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100-50}{100+50} = 0.\overline{33}$$

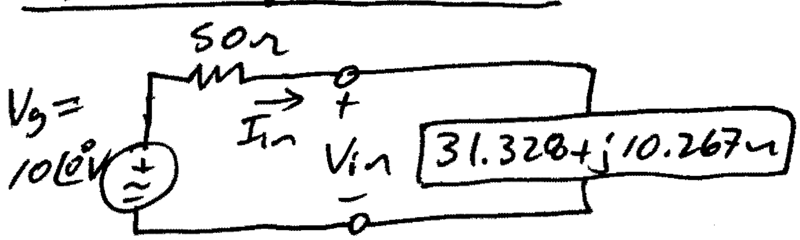
$$(2.90)\quad \Gamma(\ell) = \Gamma_{in} = \Gamma_L e^{-2\gamma\ell} = 0.\overline{33}\, e^{-2(0.069+j2\pi)1.8}$$

$$\Gamma_{in} = 0.25994\angle 144^\circ$$

$$(\text{notes})\quad Z_{in} = Z_0\frac{1+\Gamma_{in}}{1-\Gamma_{in}} = 50\,\frac{1+0.2599\angle 144^\circ}{1-0.2599\angle 144^\circ}$$

$$Z_{in} = 31.328 + j\,10.267\ \Omega$$

Equiv. input circuit



$$I_{in} = \frac{10\angle 0}{50 + (31.33 + j10.27)} = 0.12199\angle -7.1951^\circ\ \text{A}$$

$$V_{in} = 10\angle 0^\circ\,\frac{31.33 + j10.27}{50 + (31.33 + j10.27)} = 4.0217\angle 10.9502^\circ\ \text{V}$$

(2.92) $P_{in} = \frac{1}{2} Re(V_{in} I_{in}^*) = \frac{1}{2} Re\{4.02\angle 10.95^\circ (0.122\angle 7.195^\circ)\}$

$$\underline{\underline{P_{in} = 0.233107\ W}}$$

(circuits) $P_{V_g} = \frac{1}{2} Re\{V_g I_{in}^*\} = \frac{1}{2} Re\{10\angle 0^\circ (0.122\angle 7.195^\circ)\}$

$$\underline{\underline{P_{V_g} = 0.60515\ W}}$$

Use (2.89a), $V(z) = V_0^+(e^{-\gamma z} + \Gamma e^{\gamma z})$ and $V_{in} = V(z=-\ell)$

to get $V_0^+ = \dfrac{4.0217\angle 10.95^\circ}{e^{+(0.0691+j2\pi)1.8} + 0.\overline{33}\, e^{-(0.0691+j2\pi)1.8}}$

$$V_0^+ = 4.4154\angle 72^\circ\ V$$

We can now compute V_L & I_L.

$V_L = V(0) = 4.4154\angle 72^\circ \left(e^{-(0.0691+j2\pi)0} + 0.\overline{33}\, e^{(0.0691+j2\pi)0}\right)$

$$V_L = 5.8872\angle 72^\circ\ V$$

$$I_L = V_L/Z_L = \frac{5.8872\angle 72^\circ}{100} = 0.058872\angle 72^\circ\ A$$

Per (2.93), $P_L = \frac{1}{2} Re\{V_L I_L^*\} = \frac{1}{2} Re\{5.8872\angle 72^\circ (0.058872\angle -72^\circ)\}$

$$\underline{\underline{P_L = 0.173296\ W}}$$

Per (2.94), $P_{loss} = P_{in} - P_L = 0.233107 - 0.173296$

$$\underline{\underline{P_{loss} = 0.059812\ W}}$$