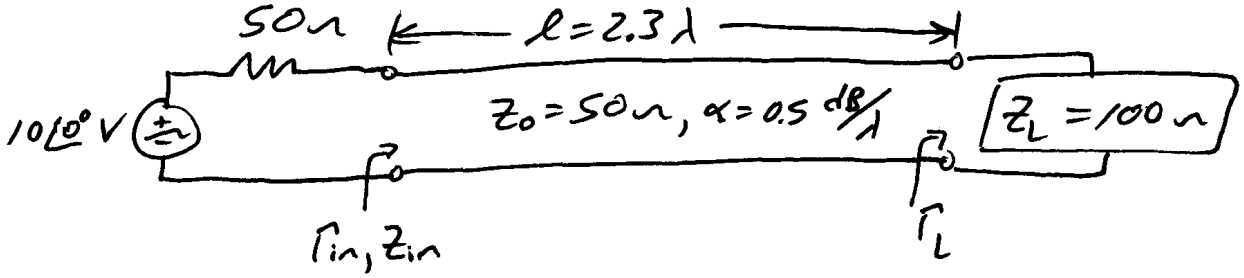
2.29 A 50 Ω transmission line is matched to a 10 V source and feeds a load $Z_L = 100\ \Omega$. If the line is 2.3λ long and has an attenuation constant $\alpha = 0.5$ dB/λ, find the powers that are delivered by the source, lost in the line, and delivered to the load.



$$\alpha = 0.5\ \frac{dB}{\lambda}\ \frac{1\,Np}{20\log_e dB} = 0.057564\ 63\ Np/\lambda$$

$$\beta = \frac{2\pi}{\lambda} \Rightarrow \gamma = \alpha + j\beta = (0.057565 + j2\pi)/\lambda$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100-50}{100+50} = 0.\overline{3}$$

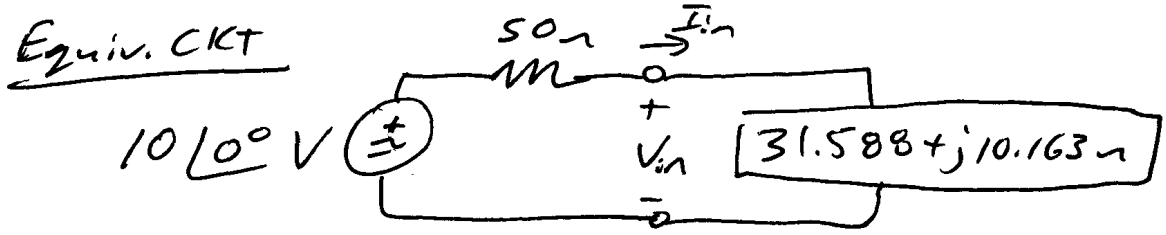
$$(2.90)\quad \Gamma_{in} = \Gamma_L e^{-2\gamma\ell} = 0.\overline{3}\, e^{-2\left(\frac{0.057565 + j2\pi}{\lambda}\right)2.3\lambda}$$

$$\Gamma_{in} = 0.2558\angle 144^\circ$$

$$Z_{in} = Z_0\frac{1+\Gamma_{in}}{1-\Gamma_{in}} = 50\,\frac{1+0.2558\angle 144^\circ}{1-0.2558\angle 144^\circ}$$

$$= 31.588 + j\,10.163\ \Omega$$

Equiv. CKT



$$I_{in} = \frac{10\angle 0^\circ}{50 + (31.588 + j10.163)} = 0.1216\angle -7.1^\circ\ A$$

$$V_{in} = 10\angle 0^\circ\,\frac{31.588 + j10.163}{50 + (31.588 + j10.163)} = 4.0359\angle 10.7347^\circ\ V$$

$$V_0^+ = \frac{V_{in}}{e^{\gamma \ell} + \Gamma_L e^{-\gamma \ell}} = \frac{4.0359 \angle 10.7347^\circ}{e^{\frac{0.057565 + j2\pi}{\lambda}(2.3\lambda)} + 0.\bar{3}\, e^{-\left(\frac{0.057565 + j2\pi}{\lambda}\right)2.3\lambda}}$$

$$V_0^+ = 4.38 \angle -108^\circ \text{ V}$$

$$P_{in} = \tfrac{1}{2} \text{Re}\{V_{in} I_{in}^*\} = \tfrac{1}{2} \text{Re}\{(4.036 \angle 10.7^\circ)(0.1216 \angle +7.1^\circ)\}$$

$$P_{in} = 0.23364 \text{ W}$$

OR (2.92)

$$P_{in} = \frac{|V_0^+|^2}{2Z_0}\left[1 - |\Gamma_{in}|^2\right] e^{2\alpha \ell} = \frac{4.38^2}{2(50)}\left\{1 - 0.2558^2\right\} e^{2(0.0576)2.3}$$

$$P_{in} = 0.23364 \text{ W} \quad \text{SAME!}$$

Per (2.89a) & (2.89b),

$$V_L = V(0) = V_0^+\left[e^0 + \Gamma_L e^0\right] = (4.38 \angle -108^\circ)\left[1 + 0.\bar{3}\right]$$

$$V_L = 5.84 \angle -108^\circ \text{ V}$$

$$I_L = V_L / Z_L = \frac{5.84 \angle -108^\circ}{100} = 0.0584 \angle -108^\circ \text{ A}$$

$$P_L = \tfrac{1}{2} \text{Re}\{V_L I_L^*\} = \tfrac{1}{2} \text{Re}\{(5.84 \angle -108^\circ)(0.0584 \angle +108^\circ)\}$$

$$P_L = 0.17052 \text{ W}$$

SAME!

OR (2.93)

$$P_L = \frac{|V_0^+|^2}{2Z_0}\left[1 - |\Gamma_L|^2\right] = \frac{4.38^2}{2(50)}\left[1 - 0.\bar{3}^2\right] = 0.17052 \text{ W}$$

$$P_{loss} = P_{in} - P_L = 0.23364 - 0.17052 \Rightarrow P_{loss} = 0.06312 \text{ W}$$

OR (2.94)

$$P_{loss} = \frac{|V_0^+|^2}{2Z_0}\left[\left(e^{2\alpha \ell} - 1\right) + |\Gamma_L|^2\left(1 - e^{-2\alpha \ell}\right)\right]$$ ↗ same