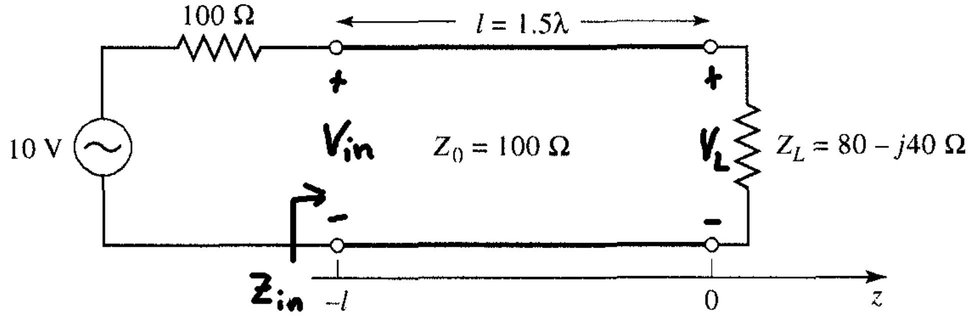
2.19 A generator is connected to a transmission line as shown in the accompanying figure. Find the voltage as a function of z along the transmission line. Plot the magnitude of this voltage for $-\ell \le z \le 0$.



Per (2.35), $\Gamma = \Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{(80 - j40) - 100}{(80 - j40) + 100}$ $\Rightarrow$ $\Gamma_L = 0.24253\angle\text{-}104.04°$.

Note that the TL is 1.5λ long, an integer multiple of λ/2 long. Therefore, the input impedance $Z_{in} = Z_L$ and $V_{in} = V_L$. By voltage division

$$V_{in} = V_L = V_g \frac{Z_{in}}{Z_g + Z_{in}} = 10\frac{80 - j40}{100 + (80 - j40)}$$ $\Rightarrow$ $V_{in} = V_L = 4.8507\angle\text{-}14.036°$.

Using (2.36a) $V(z) = V_0^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$, we can solve for V_0^+ at $z = 0$ as

$$V_0^+ = \frac{V(0)}{1+\Gamma} = \frac{V_L}{1+\Gamma} = \frac{4.8507\angle -14.036°}{1 + 0.24253\angle -104.04°}$$ $\Rightarrow$ $V_0^+ = 5\angle 0°$.

Using $\beta = 2\pi/\lambda$, we can now express the phasor voltage as

$$V(z) = 5\left(e^{-j2\pi z} + (0.24253\angle -104.04°)e^{j2\pi z}\right) \text{ where } -1.5\lambda \le z \le 0.$$

As shown below, these equations were implemented in MathCAD to get the desired plot of $|V(z)|$ for $-1.5\lambda \le z \le 0$.

As expected, per (2.40a), $V_{max} = |V_0^+|\,(1 + |\Gamma|) = 5\,(1 + 0.24253) = 6.213$ V.

As expected, per (2.40b), $V_{min} = |V_0^+|\,(1 - |\Gamma|) = 5\,(1 - 0.24253) = 3.787$ V.

$l\lambda := 1.5$ $ZL := 80 - j\cdot 40 \ \Omega$ $Z0 := 100 \ \Omega$ $Zg := 100 \ \Omega$ $Vg := 10 \ V$

$Zin := ZL$

(2.35) $\Gamma L := \dfrac{ZL - Z0}{ZL + Z0}$ $|\Gamma L| = 0.24254$ $\arg(\Gamma L)\cdot\dfrac{180}{\pi} = -104.0362$ deg

$Vin := Vg\cdot\dfrac{Zin}{Zg + Zin}$ $|Vin| = 4.85071 \ V$ $\arg(Vin)\cdot\dfrac{180}{\pi} = -14.0362$ deg

(2.36a) $V0p := \dfrac{Vin}{1 + \Gamma L}$ $V0p = 5 \ V$

(2.40a) $Vmax := V0p\cdot(1 + |\Gamma L|)$ $Vmax = 6.2127 \ V$

(2.40b) $Vmin := V0p\cdot(1 - |\Gamma L|)$ $Vmin = 3.7873 \ V$

$n := 0..100$ $z_n := -1.5 + \dfrac{n\cdot 1.5}{100}$ (2.36a) $V(z) := V0p\cdot\left(e^{-j\cdot 2\cdot\pi\cdot z} + \Gamma L\cdot e^{j\cdot 2\cdot\pi\cdot z}\right)$