2.17 For a purely reactive load impedance of the form $Z_L = jX$, show that the reflection coefficient magnitude $|\Gamma|$ is always unity. Assume that the characteristic impedance Z_0 is real.

Per (2.35), the reflection coefficient is

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX - Z_0}{jX + Z_0}.$$

For complex numbers, multiplying a complex number by its complex conjugate yields the magnitude squared of the complex number. Therefore,

$$\begin{aligned} |\Gamma|^2 &= \Gamma\Gamma^* \\ &= \left(\frac{jX - Z_0}{jX + Z_0}\right)\left(\frac{-jX - Z_0}{-jX + Z_0}\right) \\ &= \frac{X^2 - Z_0(jX) + Z_0(jX) + Z_0^2}{X^2 + Z_0(jX) + Z_0(-jX) + Z_0^2} \end{aligned}$$

$$|\Gamma|^2 = \frac{X^2 + Z_0^2}{X^2 + Z_0^2} = 1$$

The magnitude of a complex number is the positive square root of its magnitude squared, i.e.,

$$|\Gamma| = \sqrt{|\Gamma|^2} = \sqrt{1}$$

$|\Gamma| = 1$ ∴