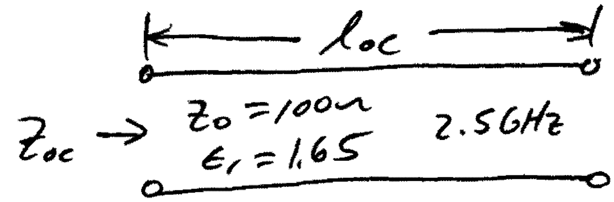
2.11 A 100 Ω transmission line has an effective dielectric constant of 1.65. Find the shortest open-circuited length of this line that appears at its input as a capacitor of 5 pF at 2.5 GHz. Repeat for an inductance of 5 nH.



For a non-magnetic TL,

$$v_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{2.9979\times10^8}{\sqrt{1.65}}$$

$$= 2.3386\times10^8 \text{ m/s}$$

$$\lambda = \frac{v_p}{f} = \frac{2.3386\times10^8}{2.5\times10^9} = 0.09335\text{ m}$$

$$\beta = 2\pi/\lambda = 67.3046 \text{ rad/m}$$

Capacitor (5pF) $\quad Z_{cap} = \dfrac{-j}{\omega C} = \dfrac{-j}{2\pi(2.5\times10^9)5\times10^{-12}}$

$$= -j12.7324\,\Omega$$

Per (2.46c), $\quad Z_{cap} = -j12.7324 = -jZ_0 \cot\beta\ell_{oc}$

$$\ell_{oc_1} = \frac{1}{67.305}\tan^{-1}\left(\frac{100}{12.7324}\right) = \underline{\underline{0.021457\text{ m}}}$$

Inductor (5nH) $\quad Z_{ind} = j\omega L = j\,2\pi(2.5\times10^9)(5\times10^{-9})$

$$= j\,78.5398\,\Omega$$

Per (2.46c), $\quad Z_{ind} = Z_{in,oc} = -j\,Z_0 \cot\beta\ell_{oc}$

$$\ell_{oc_2} = \frac{1}{\beta}\tan^{-1}\left(\frac{-Z_0}{X_{ind}}\right) = \frac{1}{67.305}\tan^{-1}\left(\frac{-100}{78.54}\right)$$

$$\ell_{oc_2} = -0.01345\text{m} + \lambda/2 = -0.01345 + \frac{0.09335}{2}$$

$$\underline{\underline{\ell_{oc_2} = 0.03323\text{ m} = 3.323\text{cm}}}$$