For the transmission line of problem 2.8, analytically determine the locations (in terms of λ) of all possible current and voltage maxima and/or minima.

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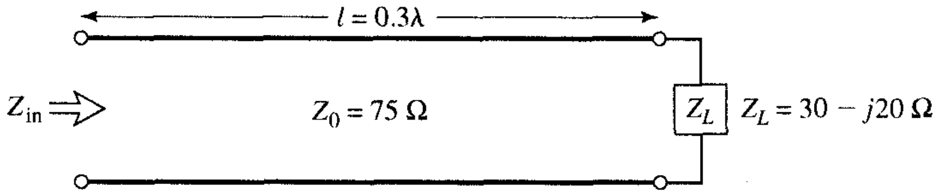
2.8 A lossless transmission line of electrical length $\ell = 0.3\lambda$ is terminated with a complex load impedance as shown in the accompanying figure. Find the reflection coefficient at the load, the SWR on the line, the reflection coefficient at the input of the line, and the input impedance to the line.



Per (2.35), $\Gamma = \Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{(30 - j20) - 75}{(30 - j20) + 75} \Rightarrow$ **$\Gamma_L = 0.46071\angle\text{-}145.2532°$.**

Using (2.10), $\beta = 2\pi / \lambda$. Therefore, $\beta\ell = 2\pi(0.3) = 0.6\,\pi = 1.88496$ rad.

Per (2.42), $\Gamma(\ell) = \Gamma_{\text{in}} = \Gamma(0)e^{-2j\beta\ell} = \Gamma_L e^{-2j\beta\ell} = (0.46071\angle -145.2532°)e^{-2j1.88496}$

$\Rightarrow$ **$\Gamma_{\text{in}} = 0.46071\angle\text{-}1.2532°$.**

Per (2.43), $Z_{\text{in}} = \dfrac{1+\Gamma e^{-2j\beta\ell}}{1-\Gamma e^{-2j\beta\ell}} Z_0 = Z_0 \dfrac{1+\Gamma_{\text{in}}}{1-\Gamma_{\text{in}}} = 75\left(\dfrac{1+0.4607\angle -1.253°}{1-0.4607\angle -1.253°}\right)$

$\Rightarrow$ **$Z_{\text{in}} = 202.99 - j\,5.19\ \Omega$.**

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Per (2.39), $|V(z)| = |V_0^+|\left|1 + |\Gamma|\, e^{j(\theta - 2\beta\ell)}\right|$ where $\ell = -z$ (distance from the load).

As covered on page 58, the voltage maxima (and current minima) occur at locations where $e^{j(\theta - 2\beta\ell)} = 1 \Rightarrow \theta - 2\beta\ell = 2\pi n$ $(n = 0, \pm 1, \pm 2, \ldots)$. Using (2.10), $\beta = 2\pi/\lambda$ and $\theta = -145.2532° = -2.53515$ rad, we get

$$\ell_{\text{Vmax}} = \ell_{\text{Imin}} = [(\theta - 2\pi n)/4\pi]\lambda = [(-2.53515 - 2\pi n)/4\pi]\lambda$$

The smallest positive value (& only point on the TL) for ℓ_{Vmax} occurs when $n = -1$,

$\ell_{\text{Vmax}} = \ell_{\text{Imin}} = [(-2.53515 + 2\pi)/4\pi]\lambda = 0.29826\,\lambda \Rightarrow$ **$z_{\text{Vmax}} = z_{\text{Imin}} = -0.29826\,\lambda$.**

As covered on p. 58, the voltage minima (& current maxima) occur at locations where $e^{j(\theta - 2\beta\ell)} = -1 \Rightarrow \theta - 2\beta\ell = \pi(2n+1)$ $(n = 0, \pm 1, \pm 2, \ldots)$. This yields-

$$\ell_{\text{Vmin}} = \ell_{\text{Imax}} = [(\theta - \pi(2n+1))/4\pi]\lambda = [(-2.53515 - \pi(2n+1))/4\pi]\lambda$$

The smallest positive value (& only point on the TL) for ℓ_{Vmin} occurs when n = -1,

$\ell_{Vmin} = \ell_{Imax} = [(-2.53515 + \pi)/4\pi]\lambda = 0.048259\,\lambda \quad \Rightarrow$ **$z_{Vmin} = z_{Imax} = -0.04826\,\lambda$.**

Verified answers using MathCad-

$l\lambda := 0.3 \quad ZL := 30 - j\cdot 20 \;\; \Omega \quad Z0 := 75 \;\; \Omega \quad Zg := 75 \;\; \Omega \quad Vg := 10 \;\; V$

$Zin := ZL$

(2.35) $\Gamma L := \dfrac{ZL - Z0}{ZL + Z0} \qquad |\Gamma L| = 0.46071 \qquad \arg(\Gamma L)\cdot\dfrac{180}{\pi} = -145.2532 \;\; deg$

$\Gamma in := \Gamma L\cdot e^{-j\cdot 2\cdot 2\cdot\pi\cdot l\lambda} \qquad |\Gamma in| = 0.46071 \qquad \arg(\Gamma in)\cdot\dfrac{180}{\pi} = -1.2532 \;\; deg$

$Zin := Z0\cdot\dfrac{1 + \Gamma in}{1 - \Gamma in} \qquad Zin = 202.99 - 5.193i \;\; \Omega$

$Vin := Vg\cdot\dfrac{Zin}{Zg + Zin} \qquad |Vin| = 7.30317 \;\; V \qquad \arg(Vin)\cdot\dfrac{180}{\pi} = -0.3953 \;\; deg$

(2.36a) $V0p := \dfrac{Vin}{e^{j\cdot 2\cdot\pi\cdot l\lambda} + \Gamma L\cdot e^{-j\cdot 2\cdot\pi\cdot l\lambda}} \qquad |V0p| = 5 \;\; V \qquad \arg(V0p)\cdot\dfrac{180}{\pi} = -108 \;\; deg$

(2.40a) $Vmax := |V0p|\cdot(1 + |\Gamma L|) \qquad Vmax = 7.3036 \;\; V$

(2.40b) $Vmin := |V0p|\cdot(1 - |\Gamma L|) \qquad Vmin = 2.6964 \;\; V$

$n := 0..100 \qquad z_n := -0.3 + \dfrac{n\cdot 0.3}{100}$ (2.36a) $V(z) := V0p\cdot\left(e^{-j\cdot 2\cdot\pi\cdot z} + \Gamma L\cdot e^{j\cdot 2\cdot\pi\cdot z}\right)$