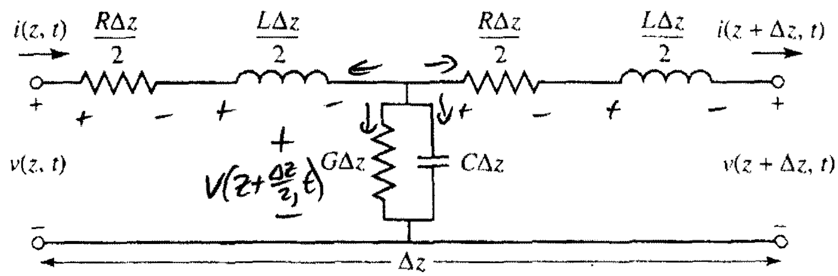
2.7 Show that the T-model of a transmission line shown in the accompanying figure also yields the telegrapher equations derived in Section 2.1.



Apply KVL to outer loop

$$-V(z,t) + i(z,t)\frac{R\Delta z}{2} + \frac{L\Delta z}{2}\frac{\partial i(z,t)}{\partial t} + i(z+\Delta z,t)\frac{R\Delta z}{2}$$

$$+ L\frac{\Delta z}{2}\frac{\partial i(z+\Delta z,t)}{\partial t} + V(z+\Delta z,t) = 0$$

⇓ re-arrange

$$-\left[\frac{V(z+\Delta z,t - V(z,t)}{\Delta z}\right] = \frac{R}{2}\left[i(z,t) + i(z+\Delta z,t)\right] + \frac{L}{2}\frac{\partial\left(i(z,t) + i(z+\Delta z,t)\right)}{\partial t}$$

Let $\Delta z \to \infty$, noting $\frac{\partial f}{\partial z} = \lim_{\Delta z \to 0}\frac{f(z+\Delta z) - f(z)}{\Delta z}$

and $\lim_{\Delta z \to 0}\left[i(z+\Delta z,t)\right] = i(z,t)$

$$+\frac{\partial V(z,t)}{\partial z} = -\frac{R}{2}\left[2i(z,t)\right] - \frac{L}{2}\frac{\partial(2i(z,t))}{\partial t}$$

Telegrapher Eq'n $\boxed{\frac{\partial V(z,t)}{\partial z} = -R\,i(z,t) - L\frac{\partial i(z,t)}{\partial t}}$ ∴ (2.2a)

Apply KCL to top middle node

$$-i(z,t) + i(z+\Delta z,t) + G\Delta z\, v\left(z+\frac{\Delta z}{2},t\right) + C\Delta z \frac{\partial v\left(z+\frac{\Delta z}{2},t\right)}{\partial t} = 0$$

$\Downarrow$ Re-arrange

$$\frac{i(z+\Delta z,t) - i(z,t)}{\Delta z} = -G\, v\left(z+\frac{\Delta z}{2},t\right) - C\frac{\partial v\left(z+\frac{\Delta z}{2},t\right)}{\partial t}$$

Again, let $\Delta z \to 0$

Telegrapher Eq'n

$$\boxed{\frac{\partial i(z,t)}{\partial t} = -G\, v(z,t) - C\frac{\partial v(z,t)}{\partial t}}$$

;, (2.2b)