A 3 GHz plane wave propagates through rexolite in the +z-direction. a) Find the phase velocity, wavelength, attenuation constant, phase constant, and intrinsic impedance. b) If the electric field has an amplitude of 6 V/m at $z = 0$ and is oriented in the x-direction, write the equation for the phasor vector electric field. c) Find the corresponding phasor vector magnetic field. d) Is rexolite a good conductor? Why or why not? Regardless, find the skin depth in rexolite at 3 GHz.

From App. G & earlier problem, $\varepsilon_r = 4.8$ & $\tan\delta = 0.00048$ for rexolite at 3 GHz. So, $\varepsilon' = 4.8\,\varepsilon_0 = 2.24896\times10^{-11}$ F/m, $\varepsilon'' = \varepsilon'\tan\delta = 1.0795\times10^{-14}$ F/m, & $\sigma = \varepsilon''\omega = 2.0348\times10^{-4}$ S/m.

a) Use equations from Table 1.1 for general lossy medium

propagation constant
$$\gamma = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon'}} = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\tan\delta}$$
$$= j2\pi 3\bullet10^{9}\sqrt{4\pi\bullet10^{-7}(2.24896\bullet10^{-11})'}\sqrt{1 - j0.00048}$$
$$\Rightarrow\ \gamma = 0.02405 + j100.206822\ 1/\text{m} = \alpha + j\beta$$

Phase velocity $v_p = \dfrac{\omega}{\beta} = \dfrac{2\pi\,3\bullet10^{9}}{100.206822}$ $\Rightarrow$ **$v_p = 1.88107\times10^{8}$ m/s**

Wavelength $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{100.2068822}$ $\Rightarrow$ **$\lambda = 0.062702$ m**

attenuation constant $\alpha = \text{Re}(\gamma) = \text{Re}(0.02405 + j100.206822)$ $\Rightarrow$ **$\alpha = 0.02405$ Np/m**

phase constant $\beta = \text{Im}(\gamma) = \text{Im}(0.02405 + j100.206822)$ $\Rightarrow$ **$\beta = 100.206822$ rad/m**

intrinsic impedance $\eta = \dfrac{j\omega\mu}{\gamma} = \dfrac{j(2\pi\,3\bullet10^{9})4\pi\bullet10^{-7}}{0.02405 + j100.206822}$ $\Rightarrow$ **$\eta = 236.3816 + j0.05673\ \Omega$**

b) Adapting (1.54) for a wave that propagates in the +z-direction, $\bar{E} = \hat{x}\,E_x = \hat{x}\,E^{+}\,e^{-\gamma z}$.

$$\Rightarrow\ \bar{E} = \hat{x}\,6\,e^{-0.02405z}\,e^{-j100.206822z}\ \text{(V/m)}$$

c) Using (1.56) for a wave that propagates in the +z-direction,

$$\bar{H} = \hat{y}\frac{-j\gamma}{\omega\mu}E^{+}e^{-\gamma z} = \hat{y}\frac{-j(0.02405 + j100.206822)}{(2\pi\,3\bullet10^{9})4\pi\bullet10^{-7}}6\,e^{-0.02405z}\,e^{-j100.206822z}$$
$$\Rightarrow\ \bar{H} = \hat{y}\,(0.025383 - j6.091844\bullet10^{-6})\,e^{-0.02405z}\,e^{-j100.206822z}\ \text{(A/m)}$$
$$= \hat{y}\,(0.025383\angle -0.01375^{\circ})\,e^{-0.02405z}\,e^{-j100.206822z}\ \text{(A/m)}$$

d) Check if $\sigma >> \varepsilon\omega$, i.e., $\tan\delta >> 1$? Since $\tan\delta = 0.00048$,

$\Rightarrow$ **rexolite is NOT a good conductor.**

Per (1.60), the skin depth is $\delta_s = 1/\alpha = 1/0.02405$ $\Rightarrow$ **$\delta_s = 41.58067$ m**