

A 93.5 MHz plane wave propagates through free space in the $-z$ -direction. a) Find the phase velocity, wavelength, phase constant, and intrinsic impedance. b) If the electric field has an amplitude of 12 V/m at $z = 0$ and is oriented in the x -direction, write the equation for the phasor vector electric field. c) Find the corresponding phasor vector magnetic field.

Free space- $\epsilon = \epsilon_0 = 8.8541878 \times 10^{-12}$ F/m and $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m

a)

$$\text{phase velocity (1.47)} \quad v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{4\pi \cdot 10^{-7} (8.8541878 \cdot 10^{-12})}} \Rightarrow \underline{v_p = 2.99792 \times 10^8 \text{ m/s}}$$

$$\text{wavelength (1.48)} \quad \lambda = \frac{2\pi}{k} = \frac{v_p}{f} = \frac{2.99792458 \cdot 10^8}{93.5 \cdot 10^6} \Rightarrow \underline{\lambda = 3.20634 \text{ m}}$$

phase/prop. constant (Table 1.1)

$$k = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\epsilon} = 2\pi \cdot 93.5 \cdot 10^6 \sqrt{4\pi \cdot 10^{-7} (8.8541878 \cdot 10^{-12})} \Rightarrow \underline{k = 1.959615 \text{ rad/m}}$$

$$\text{intrinsic impedance (Table 1.1)} \quad \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{4\pi \cdot 10^{-7}}{8.8541878 \cdot 10^{-12}}} \Rightarrow \underline{\eta = 376.7303 \Omega}$$

b) Per (1.45), $E_x(z) = E^+ e^{-jkz} + E^- e^{jkz}$. As the wave only propagates in the $-z$ -direction, we drop the first term. We are given $E_x(z=0) = 12 \text{ V/m} = E^- e^0 \Rightarrow E^- = 12 \text{ V/m}$.

Therefore, we get

$$\Rightarrow \underline{\bar{E} = \hat{x} 12 e^{j1.9596z} \text{ (V/m)}}$$

$$\text{c) Using (1.76), } \bar{H} = \frac{1}{\eta_0} (\hat{n} \times \bar{E}) = \frac{1}{376.7303} (-\hat{z} \times \hat{x} 12 e^{j1.9596z})$$

$$\Rightarrow \underline{\bar{H} = -\hat{y} 0.031853 e^{j1.9596z} \text{ (A/m)}}$$