Region 1 ($z > 0$) is air (μ_0, ε_0) while region 2 ($z < 0$) is a ferrite ($16\mu_0$, $10\varepsilon_0$). The electric field is $100\hat{a}_x - 80\hat{a}_y - 90\hat{a}_z$ (V/m) at $z = 0^+$. a) Find the electric χ_e and magnetic χ_m susceptibilities in both regions. b) Find the electric flux density and polarization vectors in region 1 at $z = 0^+$. c) Determine the electric field, electric flux density, and polarization vectors in region 2 at $z = 0^-$.

a) From notes, (1.17), and (1.25), the electric χ_e and magnetic χ_m susceptibilities are-

Region 1 air: $\chi_{e,\text{air}} = \varepsilon_{r,\text{air}} - 1 = 1 - 1 \quad \Rightarrow \quad \underline{\boldsymbol{\chi_{e,\text{air}} = 0}}$ and

$\chi_{m,\text{air}} = \mu_{r,\text{air}} - 1 = 1 - 1 \quad \Rightarrow \quad \underline{\boldsymbol{\chi_{m,\text{air}} = 0}}$.

Region 2 ferrite: $\chi_{e,\text{fer}} = \varepsilon_{r,\text{fer}} - 1 = 10 - 1 \quad \Rightarrow \quad \underline{\boldsymbol{\chi_{e,\text{fer}} = 9}}$ and

$\chi_{m,\text{fer}} = \mu_{r,\text{fer}} - 1 = 16 - 1 \quad \Rightarrow \quad \underline{\boldsymbol{\chi_{m,\text{fer}} = 15}}$.

b) Per (1.17), the electric flux density vector is $\overline{D} = \varepsilon\overline{E}$. Therefore, in region 1, we get-

$$\overline{D}_{\text{air}} = \overline{D}_1 = 8.8541878 \times 10^{-12}\left(100\hat{a}_x - 80\hat{a}_y - 90\hat{a}_z\right)$$

$$\Rightarrow \quad \underline{\overline{D}_1 = 885.419\hat{a}_x - 708.335\hat{a}_y - 796.877\hat{a}_z \ \ (\text{pC/m}^2)}.$$

Per (1.16), the polarization vector is $\overline{P}_e = \varepsilon_0\chi_e\overline{E}$. Therefore, in region 1, we get-

$$\overline{P}_{e,\text{air}} = \overline{P}_{e,1} = 8.8541878 \times 10^{-12}(0)\left(100\hat{a}_x - 80\hat{a}_y - 90\hat{a}_z\right) \quad \Rightarrow \quad \underline{\overline{P}_{e,1} = 0}.$$

c) Since no surface charge is mentioned as being applied, we assume the $\rho_s = 0$. Therefore, we can use (1.38a) $\hat{n} \bullet \overline{D}_1 = \hat{n} \bullet \overline{D}_2$ and (1.38c) $\hat{n} \times \overline{E}_1 = \hat{n} \times \overline{E}_2 \Rightarrow \overline{E}_{1,t} = \overline{E}_{2,t}$ where $\hat{n} = \hat{a}_z$. The $\hat{a}_x$ and $\hat{a}_y$ directions are tangential to boundary.

$$\hat{a}_z \bullet \overline{D}_1 = \hat{a}_z \bullet \left(885.419\hat{a}_x - 708.335\hat{a}_y - 796.877\hat{a}_z\right) = -796.877 \ (\text{pC/m}^2) = D_{2,z} = \hat{a}_z \bullet \overline{D}_2$$

and, per (1.28a), $D_{2,z} = \varepsilon_2 E_{2,z} \Rightarrow E_{2,z} = D_{2,z} / \varepsilon_2$. Therefore, we get the normal component of the electric field in region 2 to be

$$E_{2,z} = -796.877 \times 10^{-12} / (10 \bullet 8.8541878 \times 10^{-12}) = -9 \ (\text{V/m}) \ \text{or} \ \overline{E}_{2,z} = -\hat{a}_z 9 \ (\text{V/m}).$$

Next, we get the tangential component of the electric field in region 2 to be

$$\overline{E}_{1,t} = 100\hat{a}_x - 80\hat{a}_y \ (\text{V/m}) = \overline{E}_{2,t}.$$

c) continued

To get the overall electric field in region 2, add the normal and tangential components

$$\underline{\overline{E}_2 = \overline{E}_{2,t} + \overline{E}_{2,n} = 100\hat{a}_x - 80\hat{a}_y - 9\hat{a}_z \text{ (V/m)}}.$$

Per (1.17), $\overline{D}_2 = \varepsilon_2 \overline{E}_2 = (10)8.8541878 \times 10^{-12}\left(100\hat{a}_x - 80\hat{a}_y - 9\hat{a}_z\right)$. Therefore, in region 2, we get-

$$\underline{\overline{D}_2 = 8.8542\hat{a}_x - 7.0834\hat{a}_y - 0.7969\hat{a}_z \text{ (nC/m}^2\text{)}}.$$

Per (1.16), the polarization vector is $\overline{P}_e = \varepsilon_0 \chi_e \overline{E}$. Therefore, in region 2, we get $\overline{P}_{e,2} = 8.8541878 \times 10^{-12}(9)\left(100\hat{a}_x - 80\hat{a}_y - 9\hat{a}_z\right)$-

$$\underline{\overline{P}_{e,2} = 7.9688\hat{a}_x - 6.3750\hat{a}_y - 0.7172\hat{a}_z \text{ (nC/m}^2\text{)}}.$$