1.2 A plane wave traveling along the x-axis in a polystyrene-filled region with $\epsilon_r = 2.54$ has an electric field given by $E_y = E_0 \cos(\omega t - kx)$. The frequency is 2.4 GHz, and $E_0 = 5.0$ V/m. Find the following: (a) the amplitude and direction of the magnetic field, (b) the phase velocity, (c) the wavelength, and (d) the phase shift between the positions $x_1 = 0.1$ m and $x_2 = 0.15$ m.

Assume polystyrene ($\varepsilon = 2.54\varepsilon_0$, $\mu = \mu_0$) is lossless as no loss tangent information is given.

a) Adapt (1.76), $\bar{H} = \frac{1}{\eta}\hat{n} \times \bar{E}$. From the '-kx' term of E_y, we know the wave propagates in the +x-direction. Per Table 1.1, $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{4\pi \bullet 10^{-7}}{2.54 \bullet 8.8541878 \bullet 10^{-12}}} = 236.3816224\,\Omega$.

$$\bar{H} = \frac{1}{236.38}\hat{x} \times \hat{y} 5e^{-jkx} = \hat{z}\, 0.02115224\, e^{-jkx} \text{ (A/m)}$$

$\Rightarrow$ $|\bar{H}| = 0.02115224$ (A/m) in $\hat{z}$ direction

b) phase velocity (1.47), $v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{4\pi \bullet 10^{-7}(2.54)8.8541878 \bullet 10^{-12}}}$

$\Rightarrow$ **$v_p = 1.88106 \times 10^8$ m/s**

c) wavelength (1.48) $\lambda = \frac{2\pi}{k} = \frac{v_p}{f} = \frac{1.88106 \bullet 10^8}{2.4 \bullet 10^9}$ $\Rightarrow$ **$\lambda = 0.078378$ m**

d) The phase constant (Table 1.1) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.078378} = 80.1656$ rad/m.

Phase shift = $k\Delta x$ = 80.1656 (0.15 - 0.1) $\Rightarrow$ **phase shift = 4.00828 rad = 229.66°**