EE 481/581 Microwave Engineering Examination #3 (Fall 2025)

Name KEY

Instructions: Place answers in indicated spaces, use notation as given in class, and **show/explain all** work for full/partial credit. Design graphs/tables used should be annotated with relevant problem # and show all markings/work with values labeled as needed. Turn-in equation sheet(s) with exam.

1) You have been tasked with designing a quadrature hybrid coupler for use in a 75 Ω system at 920 MHz. It is to be implemented using stripline on a board where the ground planes are separated by 4 mm with a dielectric ($\varepsilon_r = 2.25$, μ_0). Fill out the table provided with the design values of the quantities labeled on the top view picture. Give all lengths/widths in millimeters (mm).

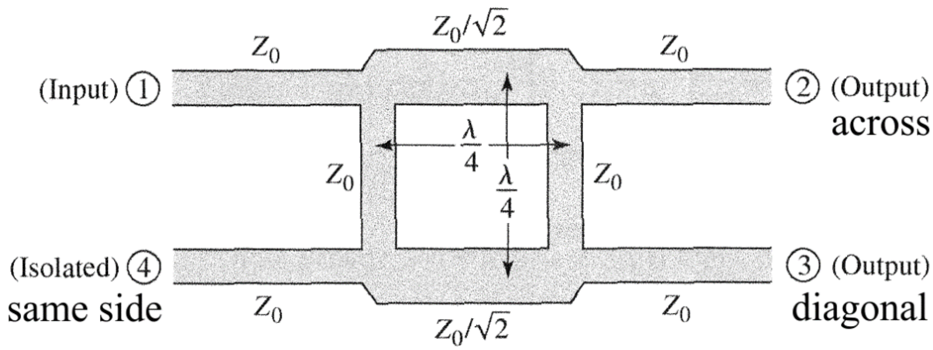


FIGURE 7.21 Geometry of a branch-line coupler.

From problem statement, Figure 7.21, and top view picture, $\underline{\boldsymbol{Z_0 = Z_{LR} = 75\,\Omega}}$.

From problem statement, $b = 4$ mm, & $\varepsilon_r = 2.25$.

Per (3.176), $v_p = c/\sqrt{\varepsilon_r} = 2.9979\times10^8/\sqrt{2.25}$ $\Rightarrow$ $\underline{v_p = 1.9986\times10^8 \text{ m/s}}$.

$\lambda = v_p/f = 1.9986\times10^8/920\times10^6 = 0.217239$ m $\Rightarrow$ $\underline{\boldsymbol{\ell_{TB} = \ell_{LR} = \lambda/4 = 54.31\text{ mm}}}$.

$\underline{\boldsymbol{Z_0 = Z_{LR} = 75\,\Omega}}$

Per (3.180a), $\dfrac{W}{b} = \begin{cases} x & \text{for } \sqrt{\varepsilon_r}Z_0 < 120\,\Omega \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\varepsilon_r}Z_0 > 120\,\Omega \end{cases}$ where (3.180b) $x = \dfrac{30\pi}{\sqrt{\varepsilon_r}Z_0} - 0.441$.

Here, $\sqrt{\varepsilon_r}Z_0 = \sqrt{2.25}\,(75) = 112.5\,\Omega < 120\,\Omega$ and $x = \dfrac{30\pi}{\sqrt{2.25}\,(75)} - 0.441 = 0.39675804$.

Using the top equation of (3.180a), $W/b = x = 0.396758$. Therefore, the land width for $Z_0 = 75\,\Omega$ is $W_0 = (x)\,b = (0.396758)\,4$ $\Rightarrow$ $\underline{\boldsymbol{W_0 = 1.5870\text{ mm}}}$.

From problem statement, Figure 7.21, and top view picture-

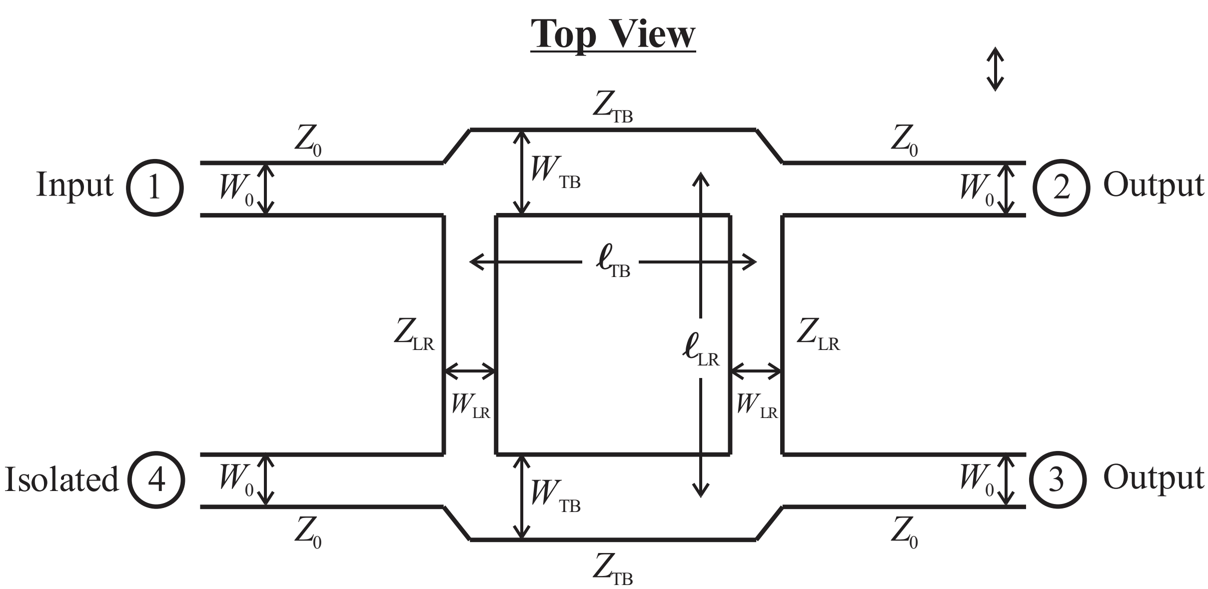
$Z_0/\sqrt{2} = Z_{TB} = 75/\sqrt{2}\,\Omega$ $\Rightarrow$ $\underline{\mathbf{Z_{TB} = 53.033\,\Omega}}$.

Per (3.180a), $\dfrac{W}{b} = \begin{cases} x & \text{for } \sqrt{\varepsilon_r}Z_0 < 120\,\Omega \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\varepsilon_r}Z_0 > 120\,\Omega \end{cases}$

where (3.180b) $x = \dfrac{30\pi}{\sqrt{\varepsilon_r}Z_0} - 0.441$.

Here, $\sqrt{\varepsilon_r}Z_{TB} = \sqrt{2.25}\,(53.033) = 79.55\ \Omega < 120\ \Omega$ and $x = \dfrac{30\pi}{79.55} - 0.441 = 0.74377$.

Using the top equation of (3.180a), $W/b = x = 0.74377$. Therefore, the land width for $Z_{TB} = 53.033\,\Omega$ is $W_{TB} = (x)\,b = (0.74377)\,4$ $\Rightarrow$ $\underline{\mathbf{W_{TB} = 2.9751\ mm}}$.



Z_0 = **75 Ω**	W_0 = **1.5870 mm**	
Z_{TB} = **53.03 Ω**	W_{TB} = **2.9751 mm**	ℓ_{TB} = **54.31 mm**
Z_{LR} = **75 Ω**	W_{LR} = **1.5870 mm**	ℓ_{LR} = **54.31 mm**

2) Design a Butterworth high-pass filter (HPF) where the desired cutoff frequency is 3 GHz with an attenuation of at least 30 dB at 1 GHz in a 50 Ω system. To begin, find the order N and immittances (g_0 to g_{N+1}) for a low-pass filter (LPF) prototype using the capacitor-first architecture (Fig. 8.25a). Sketch the LPF prototype in the box provided with all components fully labeled. Then, do the calculations necessary to impedance and frequency scale as well as transform the LPF prototype to the desired HPF. Sketch the HPF in the box provided with all components fully labeled.

Since ω is at $\omega_c / 3$ for the HPF, calculate normalized frequency for LPF prototype at $3\omega_c$, i.e., $|\omega/\omega_c| - 1 = |3\omega_c/\omega_c| - 1 = 2$. From Figure 8.26, we see that an LP prototype filter of order $\underline{N = 4}$ is needed to meet the 30 dB attenuation specification.

From Table 8.3, we get the immittances:

$\underline{g_0 = g_5 = 1}$, $\underline{g_1 = g_4 = 0.7654}$, **and** $\underline{g_2 = g_3 = 1.8478}$.

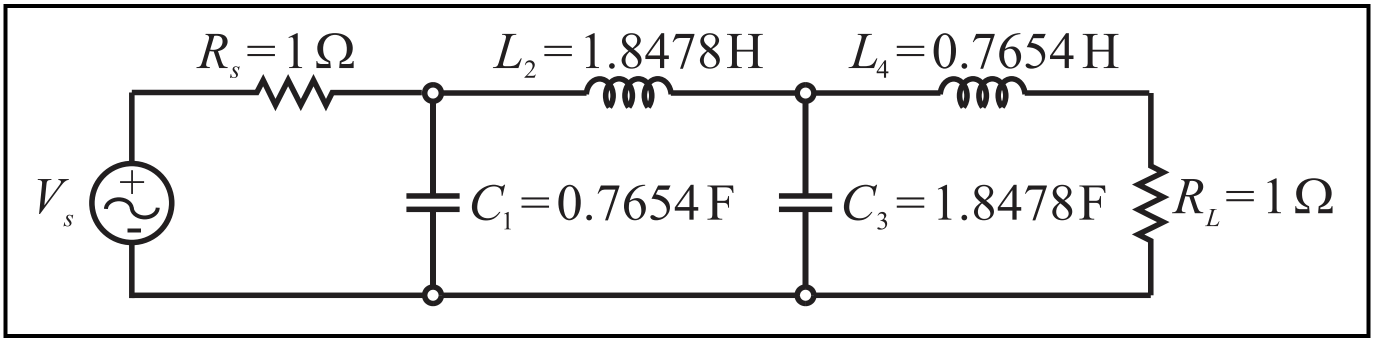
Considering Fig. 8.25a, draw the LPF protype with the following component values-

$\underline{g_0 = R_s = 1\ \Omega}$, $\underline{g_1 = C_1 = 0.7654\ \text{F}}$, $\underline{g_2 = L_2 = 1.8478\ \text{H}}$,

$\underline{g_3 = C_3 = 1.8478\ \text{F}}$, $\underline{g_4 = L_4 = 0.7654\ \text{H}}$, **and** $\underline{g_5 = R_L = 1\ \Omega}$.

$N = \underline{4}$ immittances $\underline{g_0 = 1}$, $\underline{g_1 = 0.7654}$, $\underline{g_2 = 1.8478}$, $\underline{g_3 = 1.8478}$, $\underline{g_4 = 0.7654}$, & $\underline{g_5 = 1}$

LPF prototype



Use the immittances & equations (8.64cd), & (8.70ab) to get the necessary scaled and transformed **shunt** inductances and **series** capacitances when $R_0 = 50\,\Omega$ and $f_c = 3$ GHz :

Per (8.64c), $R_S' = R_0\, g_0 = 50(1)$ $\Rightarrow$ **$R_s' = 50\,\Omega$.**

Per (8.70b), $L_1' = \dfrac{R_0}{\omega_c C_1} = \dfrac{R_0}{\omega_c g_1} = \dfrac{50}{(2\pi)3\times10^9(0.7654)}$ $\Rightarrow$ **$L_1' = 3.4656$ nH.**

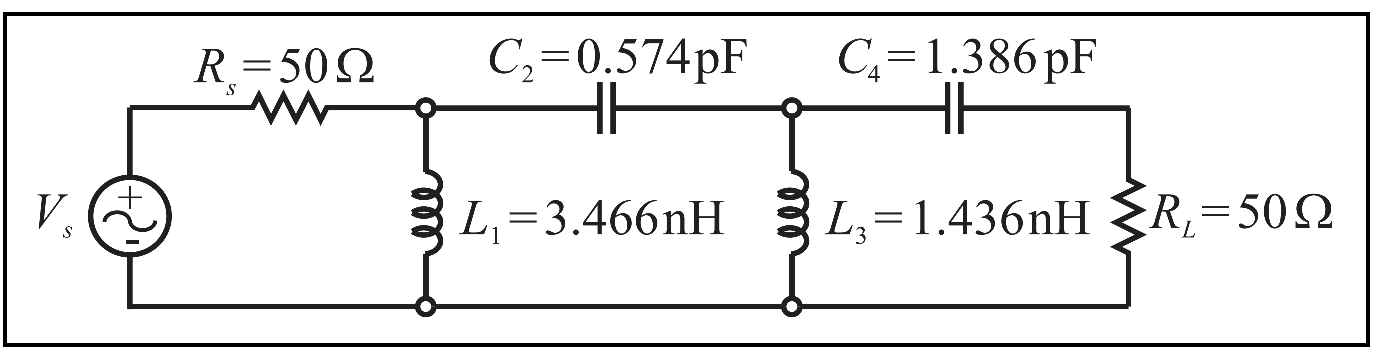
Per (8.70a), $C_2' = \dfrac{1}{R_0\,\omega_c L_2} = \dfrac{1}{R_0\,\omega_c\, g_2} = \dfrac{1}{50(2\pi)3\times10^9(1.8478)}$ $\Rightarrow$ **$C_2' = 0.5742$ pF.**

Per (8.70b), $L_3' = \dfrac{R_0}{\omega_c C_3} = \dfrac{R_0}{\omega_c g_3} = \dfrac{50}{(2\pi)3\times10^9(1.8478)}$ $\Rightarrow$ **$L_3' = 1.4355$ nH.**

Per (8.70a), $C_4' = \dfrac{1}{R_0\,\omega_c L_4} = \dfrac{1}{R_0\,\omega_c\, g_4} = \dfrac{1}{50(2\pi)3\times10^9(0.7654)}$ $\Rightarrow$ **$C_2' = 1.3862$ pF.**

Per (8.64d), $R_L' = R_0\, R_L = R_0\, g_5 = 50(1)$ $\Rightarrow$ **$R_L' = 50\,\Omega$.**

HPF



3) Design a coupled-line directional coupler with a coupling factor of 18.06 dB for use in a 39 Ω system at 1.25 GHz. It is to be implemented using edge-coupled striplines on a board where the ground planes are separated by 3 mm with a dielectric ($\varepsilon_r = 2.89$, μ_0). First, determine the unitless coupling factor C as well as the even-mode Z_{0e} and odd-mode Z_{0o} impedances. Next, find the width W_{CL}, spacing S_{CL}, and length ℓ_{CL} of the coupled lines. Also, find the width W_0 of the input/output feeding striplines. Add these values to the top view picture of the coupler. Give all lengths/widths in millimeters (mm).

Per (7.20a), $C = 10^{-CdB/20} = 10^{-18.06/20}$ $\Rightarrow$ $\underline{\boldsymbol{C = 0.125026}}$

Per (7.87a), $Z_{0e} = Z_0\sqrt{\dfrac{1+C}{1-C}} = 39\sqrt{\dfrac{1+0.125026}{1-0.125026}}$ $\Rightarrow$ $\underline{\boldsymbol{Z_{0e} = 44.223\ \Omega}}$

Per (7.87a), $Z_{0o} = Z_0\sqrt{\dfrac{1-C}{1+C}} = 39\sqrt{\dfrac{1-0.125026}{1+0.125026}}$ $\Rightarrow$ $\underline{\boldsymbol{Z_{0o} = 34.394\ \Omega}}$

To use edge-coupled stripline graph (Figure 7.29), we need

$\sqrt{\varepsilon_r}\, Z_{0e} = \sqrt{2.89}\,(44.223) = 75.18\ \Omega$ and $\sqrt{\varepsilon_r}\, Z_{0o} = \sqrt{2.89}\,(34.394) = 58.47\ \Omega$.

From Figure 7.29), we get $S/b = 0.23$ and $W/b = 0.87$

$S_{CL} = S/b\ (b) = 0.23\ (3)$ $\Rightarrow$ $\underline{\boldsymbol{S_{CL} = 0.69\text{ mm}}}$

$W_{CL} = W/b\ (b) = 0.87\ (3)$ $\Rightarrow$ $\underline{\boldsymbol{W_{CL} = 2.61\text{ mm}}}$

From problem statement, $b = 3$ mm, & $\varepsilon_r = 2.89$.

Per (3.176), $v_p = c / \sqrt{\varepsilon_r} = 2.9979\times10^8 / \sqrt{2.89}$ $\Rightarrow$ $\underline{v_p = 1.7635 \times 10^8\text{ m/s}}$.

$\lambda = v_p / f = 1.7635\times10^8 / 1.25\times10^9 = 0.14107765\text{ m} \Rightarrow \underline{\boldsymbol{\ell_{CL} = \lambda / 4 = 35.27\text{ mm}}}$

$Z_0 = 39\ \Omega$

From problem statement, b = 3 mm, & $\varepsilon_r = 2.89$.

Per (3.180a), $\dfrac{W}{b} = \begin{cases} x & \text{for } \sqrt{\varepsilon_r}\,Z_0 < 120\ \Omega \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\varepsilon_r}\,Z_0 > 120\ \Omega \end{cases}$

where (3.180b) $x = \dfrac{30\pi}{\sqrt{\varepsilon_r}\,Z_0} - 0.441$.

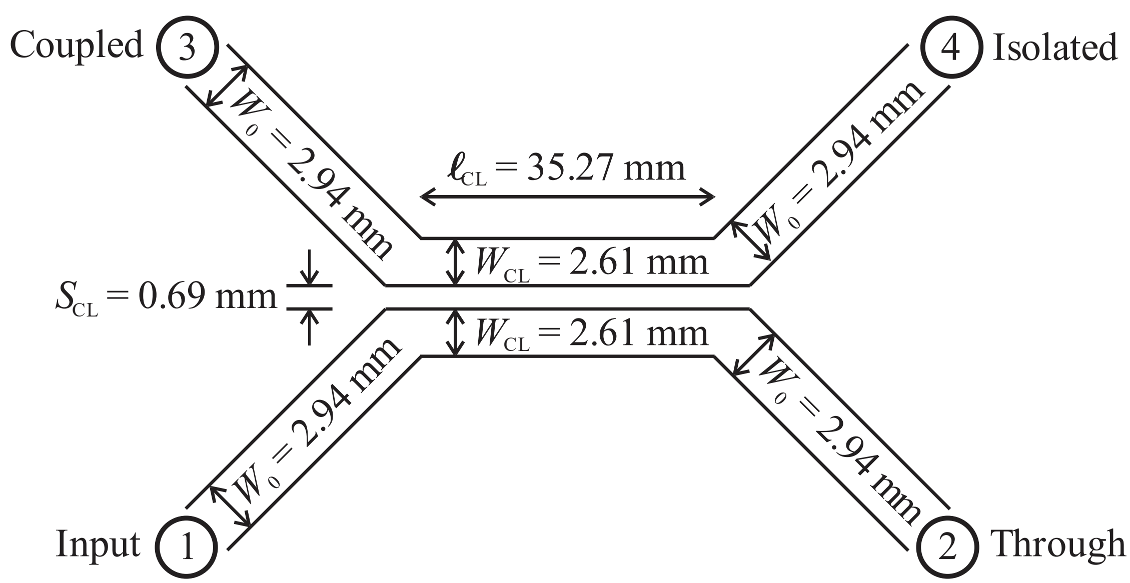
Here, $\sqrt{\varepsilon_r}\,Z_0 = \sqrt{2.89}\,(39)$ = 66.3 Ω < 120 Ω & $x = \dfrac{30\pi}{\sqrt{2.89}\,(39)} - 0.441 = 0.980535$.

Using the top equation of (3.180a), $W/b = x = 0.980535$. Therefore, the land width for $Z_0 = 39\ \Omega$ is $W_0 = (x)\,b = (0.980535\ )\,3$ $\Rightarrow$ **$W_0 = 2.942$ mm**.

C = **0.125026** Z_{0e} = **44.223 Ω** Z_{0o} = **34.394 Ω**

W_{CL} = **2.61 mm** S_{CL} = **0.69 mm** ℓ_{CL} = **35.27 mm** W_0 = **2.94 mm**

Top View



Problem 3

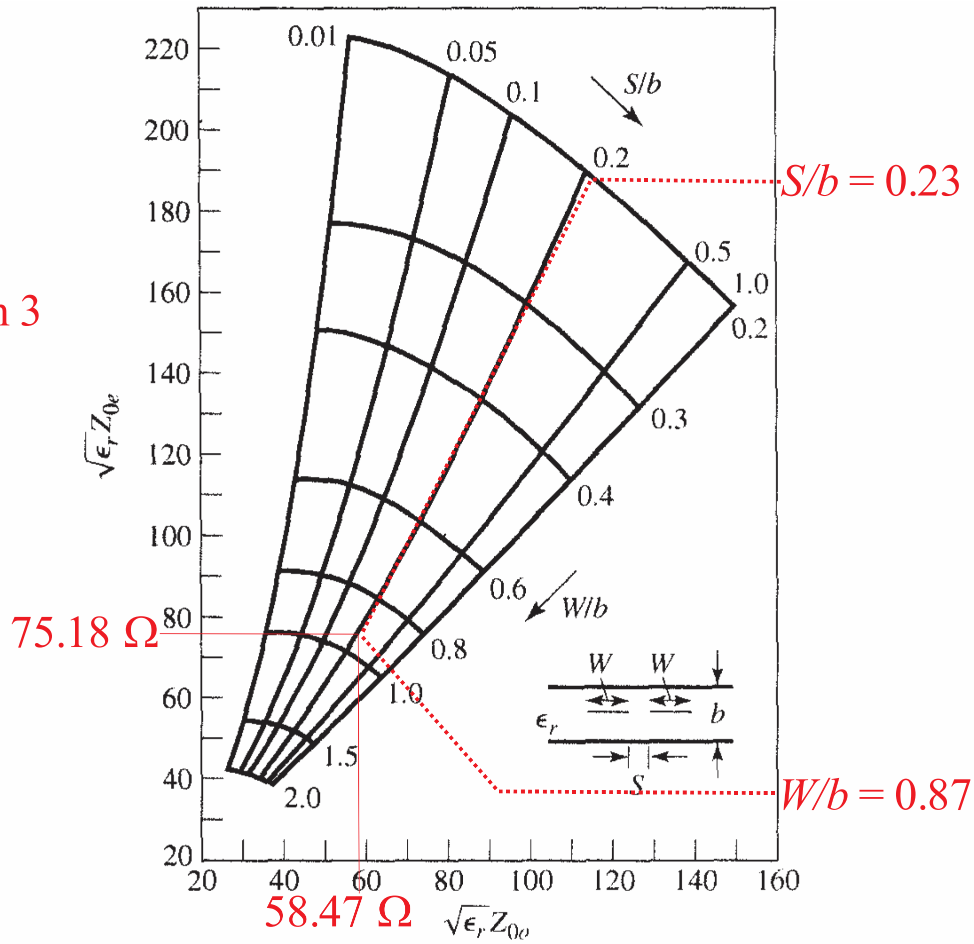


FIGURE 7.29 Normalized even- and odd-mode characteristic impedance design data for symmetric edge-coupled striplines.

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Problem 2

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

Problem 2

30 dB

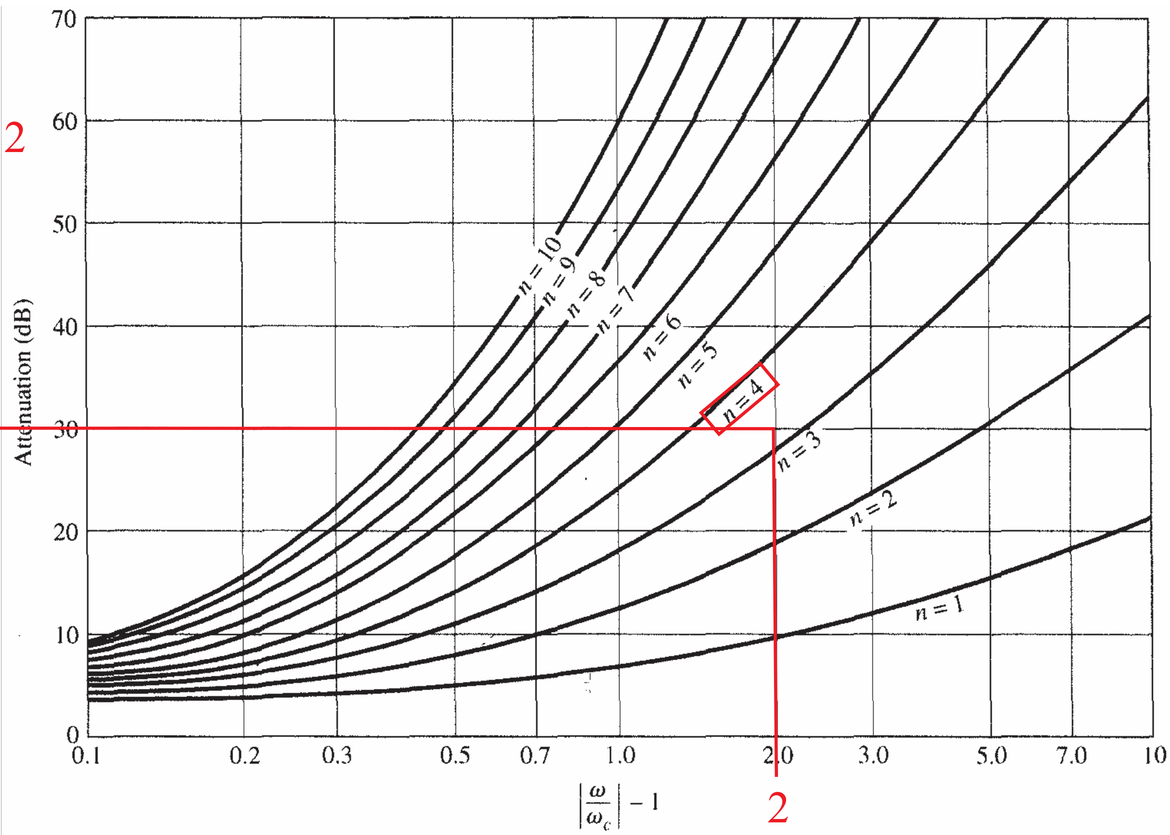


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FIGURE 8.26 Attenuation versus normalized frequency for maximally flat filter prototypes.

Adapted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.