

EE 481/581 Microwave Engineering

Quiz #5 (Fall 2024)

Name Key A

Instructions: Open book & notes. Place answers in indicated spaces and show all work for credit.

Given an air-filled 9 cm x 4 cm brass waveguide, find the two lowest cutoff frequencies and corresponding mode(s). At 1.8 GHz, find the phase velocity and wave impedance of the TE₁₀ mode.

$b = 4 \text{ cm}$  $a = 9 \text{ cm}$ $\sim \text{brass}$

$$(3.84) f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

for $a > b$ TE₁₀ is lowest mode

$$(3.85) f_{c,10} = \frac{1}{2a\sqrt{\mu\epsilon}} = \frac{c}{2a}$$

$$= \frac{2.9979 \times 10^8}{2(0.09)} = \underline{1.6655 \text{ GHz}}$$

Since $a > 2b$, the next lowest mode is TE₂₀

$$f_{c,20} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \frac{2\pi}{a} = \frac{1}{a\sqrt{\mu\epsilon}} = 2f_{c,10} = \underline{3.331 \text{ GHz}}$$

$$(3.91) \beta_{10} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2} = \sqrt{\omega^2\mu\epsilon - \left(\frac{\pi}{a}\right)^2}$$

$$= \sqrt{(2\pi \cdot 1.8 \times 10^9)^2 \cdot 4\pi \times 10^{-7} (8.8542 \times 10^{-12}) - \left(\frac{\pi}{0.09}\right)^2}$$

$$= 14.308103 \text{ rad/m}$$

$$(3.88) v_p = \frac{\omega}{\beta} = \frac{2\pi \cdot 1.8 \times 10^9}{14.3081} = \underline{7.9044255 \times 10^8 \text{ m/s} > c!}$$

$$(3.86) Z_{TE} = \frac{k\eta}{\beta} = \frac{\omega\eta_0}{\beta_{10}} = \frac{2\pi \cdot 1.8 \times 10^9}{2.9979 \times 10^8} \frac{376.7303}{14.3081}$$

$$= \underline{993.308 \Omega}$$

Mode(s)	Cutoff frequency
TE ₁₀	1.6655 GHz
TE ₂₀	3.331 GHz

phase velocity = $7.9044 \times 10^8 \text{ m/s}$

wave impedance = 993.3Ω