

## EE 481/581 Microwave Engineering Examination #3 (Fall 2024)

Name Key

Instructions: Place answers in indicated spaces, use notation as given in class, and show/explain all work (including that on design figures/tables) for credit. Turn-in equation sheet(s) with exam.

- 1) For a  $75 \Omega$  system connected port 1, design a Wilkinson power divider to operate at 1.499 GHz where 3/5 of the power input into Port 1 goes to Port 2 and 2/5 of the power input into Port 1 goes to Port 3. First, assuming all the transmission lines use an air dielectric, find the phase velocity and wavelength. Then, find and fill-in all values indicated on the figure.

$$\text{For air dielectric, } V_p = c = 2.9979 \times 10^8 \text{ m/s}$$

$$\lambda = \frac{V_p}{f} = \frac{2.9979 \times 10^8}{1.499 \times 10^9} = 0.2000 \text{ m}$$

$$\beta_{332}, k^2 = \frac{\rho_3}{\rho_2} = \frac{0.4\rho_1}{0.6\rho_1} = \gamma_3 = 0.66 \Rightarrow k = 0.8165$$

$$(7.37a) Z_{03} = Z_0 \sqrt{\frac{1+k^2}{k^3}} = 75 \sqrt{\frac{1+\gamma_3}{0.8165^3}} = 131.2363 \Omega$$

$$(7.37b) Z_{02} = k^2 Z_{03} = \gamma_3 (131.2363) = 87.4909 \Omega$$

$$(7.37c) R = Z_0 (k + \frac{1}{k}) = 75 (0.8165 + \frac{1}{0.8165}) = 153.0931 \Omega$$

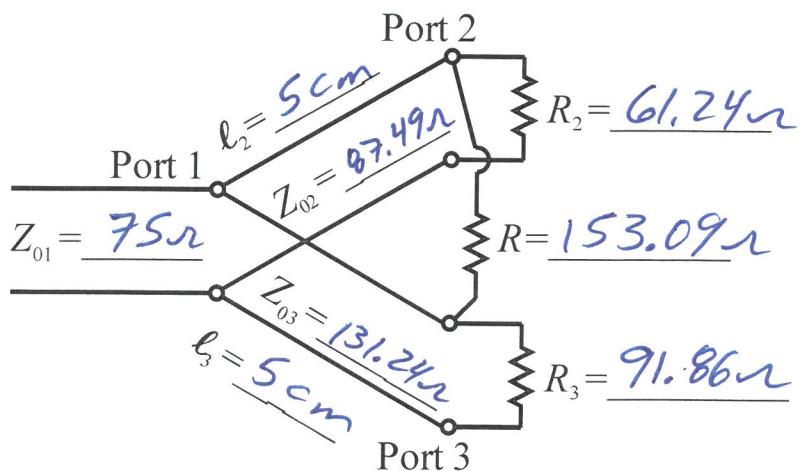
$$\text{Per Fig 7.8, the arm lengths are } \frac{\lambda}{4} \Rightarrow l_2 = l_3 = \frac{0.2}{4} = 0.05 \text{ m}$$

$$\text{Per Fig 7.13, } Z_{01} = Z_0 = 75 \Omega$$

$$R_2 = Z_0 k = 75 (0.8165) = 61.2372 \Omega$$

$$R_3 = Z_0 / k = 75 / 0.8165 = 91.855865 \Omega$$

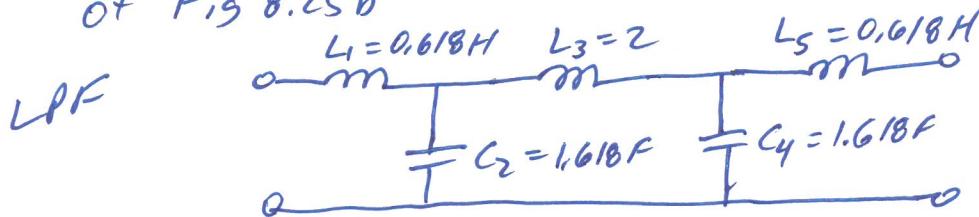
$$\text{phase velocity} = 2.9979 \times 10^8 \text{ m/s} \quad \text{wavelength} = 0.2 \text{ m}$$



- 2) Design the lowest-order, maximally flat, lumped-element, **high-pass filter (HPF)** with a cutoff frequency of 2 GHz having at least 28 dB of attenuation at 1 GHz for use in a  $75 \Omega$  system. First, determine the order  $N$  and the low-pass prototype immittance values  $g_0$  to  $g_{N+1}$ . Then, find the component values and draw a fully labeled sketch of the HPF. Minimize the use of inductors in the final design.

For LPF prototype,  $| \frac{w}{w_c} | - 1 = | \frac{z}{j} | - 1 = 1$ . Plot on Fig 8.26 along w/ 28 dB attenuation. Need  $N=5$ ,

Per HPF transformation  $L \rightarrow C$  and  $C \rightarrow L$ . To minimize inductors in HPE, choose LPF architecture of Fig 8.25b



$$\underline{R_o = 75 \Omega} + \underline{\omega_c = 2\pi(2 \times 10^9) \text{ rad/s}}$$

$$N = \underline{5}$$

immittances:  $\underline{g_0 = 1}, \underline{g_1 = 0.618}, \underline{g_2 = 1.618}, \underline{g_3 = 2}, \underline{g_4 = 1.618}, \underline{g_5 = 0.618}$   
 $\underline{+ g_6 = 1}$

Per (8.70a), the series capacitors which replace the series inductors are  $C_{LC}' = \frac{1}{R_o \omega_c L_{LC}}$

$$C_1' = C_5' = \frac{1}{75(2\pi)2 \times 10^9(0.618)} = \underline{1.716882 \mu F}$$

$$C_3' = \frac{1}{75(2\pi)2 \times 10^9(2)} = \underline{0.5305165 \mu F}$$

Per (8.70b), the shunt inductors which replace the shunt capacitors are  $L_{LC}' = \frac{R_o}{\omega_c C_{LC}}$

$$L_2' = L_4' = \frac{75}{2\pi(2 \times 10^9)(1.618)} = \underline{3.688696 nH}$$

$$R_S = R_L = 75 \Omega = \underline{75 \Omega} \quad \text{per (8.64c) \& (8.64d)}$$

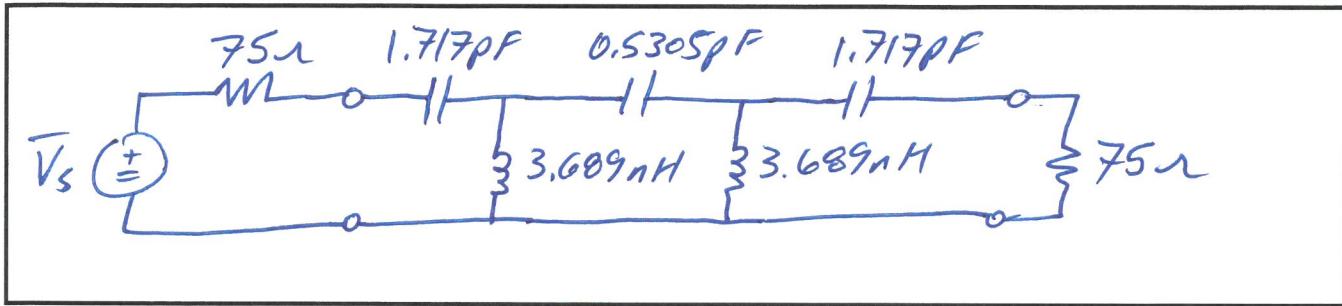


TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ( $g_0 = 1$ ,  $\omega_c = 1$ ,  $N = 1$  to 10)

$N$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$	$g_{11}$
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

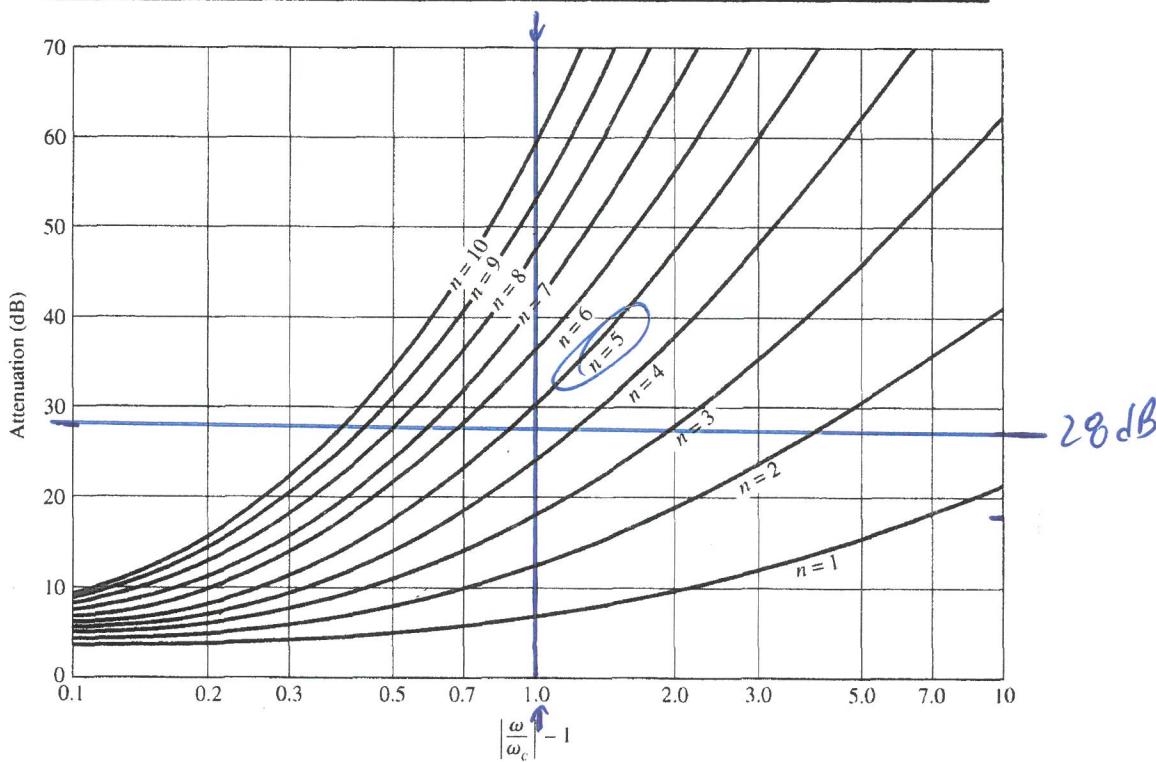


FIGURE 8.26 Attenuation versus normalized frequency for maximally flat filter prototypes.

Adapted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

- 3) A directional coupler has the  $[S]$ -matrix shown below. In **decibels**, determine the return loss, coupling factor, directivity, isolation, and insertion loss.

$$[S] = \begin{bmatrix} 0.08\angle 16^\circ & 0.96\angle 88^\circ & 0.2\angle 180^\circ & 0.006\angle 90^\circ \\ 0.96\angle 88^\circ & 0.08\angle 16^\circ & 0.006\angle 90^\circ & 0.2\angle 180^\circ \\ 0.2\angle 180^\circ & 0.006\angle 90^\circ & 0.08\angle 16^\circ & 0.96\angle 88^\circ \\ 0.006\angle 90^\circ & 0.2\angle 180^\circ & 0.96\angle 88^\circ & 0.08\angle 16^\circ \end{bmatrix}$$

Per (7.38),  $RL = -20 \log |F| = -20 \log |S_{11}| \quad S_{11} = 0.08 \angle 16^\circ$   
 $= -20 \log (0.08) = \underline{21.9382 \text{ dB}}$

Coupling (7.20a),  $C = -20 \log \beta = -20 \log |S_{13}| \quad S_{13} = 0.2 \angle 80^\circ$   
 $= -20 \log 0.2 = \underline{13.9799 \text{ dB}}$

Directivity (7.20b),  $D = 20 \log \frac{\beta}{|S_{14}|} = 20 \log \frac{|S_{13}|}{|S_{14}|} \quad S_{14} = 0.006 \angle 90^\circ$   
 $= 20 \log \frac{0.2}{0.006} = \underline{30.4576 \text{ dB}}$

Isolation (7.20c),  $I = -20 \log |S_{14}|$   
 $= -20 \log 0.006 = \underline{44.437975 \text{ dB}}$

Insertion (7.20d),  $IL = -20 \log |S_{12}| \quad S_{12} = 0.96 \angle 88^\circ$   
 $\text{loss} = -20 \log 0.96 = \underline{0.3546 \text{ dB}}$

return loss = 21.938 dB      coupling factor = 13.979 dB      directivity = 30.458 dB  
isolation = 44.437 dB      insertion loss = 0.355 dB

- 4) Design a third-order, 0.5 dB equal-ripple, lumped-element, **low-pass filter** prototype using the architecture of Figure 8.25b (use Thevenin equivalent source) and draw a fully labeled sketch in the appropriate box. Then, use Richards's transformation to implement the filter using series/shunt open/short-circuit stubs and draw a fully labeled sketch in the appropriate box. The lengths  $\ell$  may be left in terms of  $\lambda$  at  $f_c$ .

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ( $g_0 = 1, \omega_c = 1, N = 1$  to 10, 0.5 dB ripple)

$N$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$	$g_{11}$
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

$$L_1 = g_1 = 1.5963 \text{ H}$$

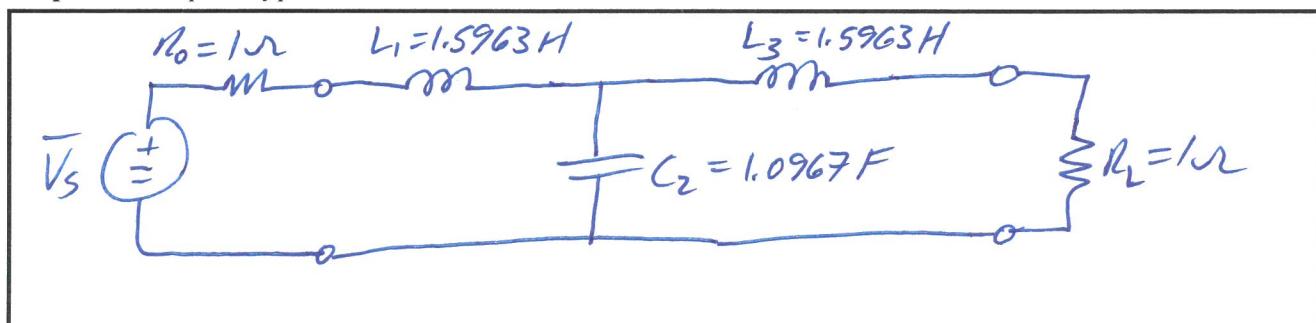
$$R_o = g_0 = 1 \Omega$$

$$C_2 = g_2 = 1.0967 \text{ F}$$

$$R_L = g_4 = 1 \Omega$$

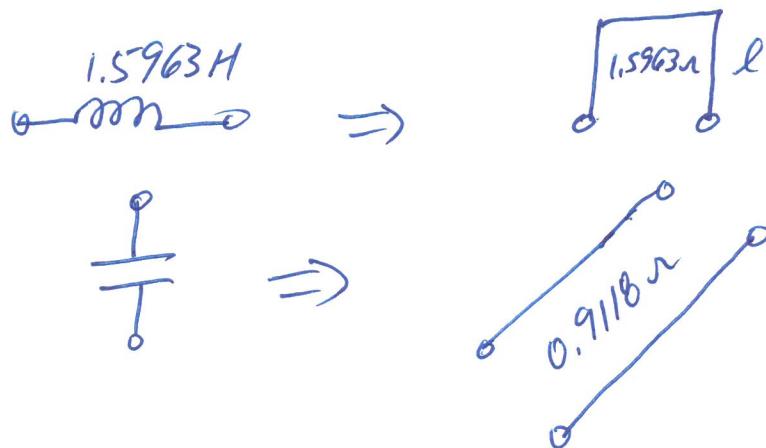
$$L_3 = g_3 = 1.5963 \text{ H}$$

low-pass filter prototype



⇒ Not doing any freq. or impedance scaling.  
 After Fig 8.34a), the series inductors become series short-circuit stubs w/  $Z_0 = L$  and length  $\ell = \lambda/8$  @  $\omega_c = 1 \text{ rad/s}$   
 Here,  $Z_0 = L = 1.5963 \Omega$

\* Per Fig 8.39 b), the shunt capacitor becomes  
 a shunt open-circuit stub w/  $Z_0 = \frac{1}{C}$   
 and length  $l = \lambda/8 @ \omega_c$ .  $Z_0 = \frac{1}{1.0967} = 0.911826 \Omega$



low-pass filter w/ stubs

