EE 481/581 Microwave Engineering Examination #1 (Fall 2024)

Name ___KEY___

Instructions: Place answers in indicated spaces, use notation as given in class, and show/explain all work for credit. All Smith charts should be fully labeled. Turn-in equation sheet(s) with exam.

1) A material has a relative permittivity of 8 and a loss tangent of 0.03 at 3 GHz. Determine the complex permittivity and effective conductivity of the material at 3 GHz. A 3 GHz plane wave propagates through this material in the +*x*-direction. Find the propagation constant, phase velocity, wavelength, skin depth, and intrinsic impedance. The electric field has an amplitude of 16 V/m at *x* = 0 and is oriented in the +*z*-direction, write the equation for the phasor vector electric field.

$\tan\delta = \frac{\sigma}{\omega\epsilon'} = 0.03$ and $\epsilon_r = \epsilon'/\epsilon_0 = 8.$

Assume $\mu = \mu_0$ (non-magnetic)

Notes: $\epsilon' = \epsilon_r\epsilon_0 = 8(8.8541878\times10^{-12}) = 70.8335\times10^{-12}$ F/m

$\epsilon'' = \epsilon'\tan\delta = 70.8335\times10^{-12}(0.03) = 2.125005\times10^{-12}$ F/m

$\epsilon_c = \epsilon' - j\epsilon'' = \underline{70.8335 - j2.125 \text{ pF/m}}$

eff. cond. $\sigma = \omega\epsilon'' = 2\pi(3\times10^9)2.125005\times10^{-12} = \underline{0.040055 \text{ S/m}}$

Table 1.1 or (1.52) $\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\sigma/\omega\epsilon}$ ← loss tangent

$= j2\pi(3\times10^9)\sqrt{4\pi\times10^{-7}(8)8.8542\times10^{-12}}\sqrt{1 - j0.03}$

$= \underline{2.667275 + j177.85835 \text{ 1/m}}$

$v_p = \omega/\beta = 2\pi(3\times10^9)/177.858 = \underline{1.05981\times10^8 \text{ m/s}}$

$\lambda = 2\pi/\beta = 2\pi/177.858 = \underline{0.035327 \text{ m}}$

$\delta_s = 1/\alpha = 1/2.667 = \underline{0.37491 \text{ m}}$

$\eta = \frac{j\omega\mu}{\gamma} = j\frac{2\pi(3\times10^9)4\pi\times10^{-7}}{2.667 + j177.86} = \underline{133.14936 + j1.99679\,\Omega}$

complex perm. = $70.8335 - j2.125$ pF/m   eff. conductivity = 0.40055 S/m

prop. const. = $2.667 + j177.858$ 1/m   phase velocity = 1.06×10^8 m/s λ = 3.533 cm

skin depth = 37.49 cm   intr. imped. = $133.15 + j1.997\,\Omega$ $\bar{E} = \hat{a}_z 16 e^{-(2.667+j177.9)x}$ (V/m)

2) A two-wire transmission line (TL) is made using a platinum-gold alloy (ε_0, μ_0, $\sigma = 1.3 \times 10^7$ S/m) for the conductors and a plastic for the insulator ($\varepsilon_r = 1.4$, μ_0, $\tan\delta = 0.003$) for operation at 4 GHz. If the conductors have diameters of 3 mm and are separated center-to-center by 12 mm respectively, find the complex permittivity of the insulator, the distributed parameters R, L, G, & C, and characteristic impedance of the transmission line.

$$\epsilon' = \epsilon_r \epsilon_0 = 1.4(8.8542\times10^{-12}) = 12.396\times10^{-12}\ \text{F/m}$$

$$\epsilon'' = \epsilon' \tan\delta = 12.396\times10^{-12}(0.003) = 0.037188\times10^{-12}\ \text{F/m}$$

$$\epsilon_c = \epsilon' - j\epsilon'' = \underline{12.396 - j0.0372\ \text{pF/m}}$$

$$(1.60)\quad \delta_s = \sqrt{\frac{2}{\omega\mu\sigma_c}} = \sqrt{\frac{2}{2\pi(4\times10^9)\,4\pi\times10^{-7}(1.3\times10^7)}} = 2.20708\times10^{-6}\ \text{m}$$

$$R_s = \frac{1}{\sigma\delta_s} = \frac{1}{1.3\times10^7(2.207\times10^{-6})} = 0.034853\ \Omega$$

Table 2.1

$$R = \frac{R_s}{\pi a} = \frac{0.034853}{\pi(3\times10^{-3}/2)} = \underline{7.396\ \Omega/\text{m}}$$

$$L = \frac{\mu_0}{\pi}\cosh^{-1}\left(\frac{d}{2a}\right) = \frac{4\pi\times10^{-7}}{\pi}\cosh^{-1}\left(\frac{12}{3}\right) = \underline{8.25375\times10^{-7}\ \text{H/m}}$$

$$G = \frac{\pi\omega\epsilon''}{\cosh^{-1}(d/2a)} = \frac{\pi(2\pi)4\times10^9(3.7188\times10^{-14})}{\cosh^{-1}(12/3)} = \underline{1.42297\times10^{-3}\ \text{S/m}}$$

$$C = \frac{\pi\epsilon'}{\cosh^{-1}(d/2a)} = \frac{\pi\, 12.396\times10^{-12}}{\cosh^{-1}(12/3)} = \underline{18.8728\times10^{-12}\ \text{F/m}}$$

$$(2.7)\quad Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{7.396 + j2\pi(4\times10^9)8.25375\times10^{-7}}{1.423\times10^{-3} + j2\pi(4\times10^9)18.873\times10^{-12}}}$$

$$\underline{Z_0 = 209.1253 + j0.276407\ \Omega}$$

ε_c = 12.396 − j0.0372 pF/m　　R = 7.396 Ω/m　　L = 825.375 nH/m

G = 1.423 mS/m　　C = 18.873 pF/m　　Z_0 = 209.1253 + j0.2764 Ω

3) A generator ($V_g = 16\angle 0°$ V, $Z_g = 300\ \Omega$) operating at 3 GHz drives a lossless TL ($300\ \Omega$, 2.1×10^8 m/s) of length 9.6 cm connected to a $60 + j120\ \Omega$ load. Using a Smith chart, find the unmatched input impedance and power delivered to the load. Then, design and sketch (fully labeled) a shunt single inductor matching network. Place the inductor as close as possible to the load. Find the matched input impedance and power delivered to the load.

$\lambda = v_p/f = \frac{2.1\times10^8}{3\times10^9} = 0.07\ m = 7\ cm \qquad \ell/\lambda = \frac{9.6}{7} = 1.3714 \to 0.3714$

$\to z_L = \frac{Z_L}{Z_0} = \frac{60+j120}{300} = 0.2 + j0.4\ \Omega/\Omega \leftarrow$ Plot on Smith Chrt

$\to$ Move 0.3714 WTG on circle of constant $|\Gamma|$ to

$z_{in,NM} = 0.205 - j0.425\ \Omega/\Omega$

$Z_{in,NM} = z_{in,NM} Z_0 = (0.205 - j0.425)(300) = \underline{61.5 - j127.5\ \Omega}$

(exact) $61.12 - j127.6\ \Omega$

$\to$ Calculate $I_{in,NM} = \frac{V_G}{Z_0 + Z_{in,NM}} = \frac{16}{300 + 61.5 - j127.5} = 0.04174\angle 19.4°\ A$

$\to$ Calculate $P_{L,NM} = P_{in,NM} = \frac{1}{2}|I_{in}|^2 Re\{Z_{in,NM}\} = 0.5(0.04174)^2\, 61.5$

$= \underline{53.57\ mW}$ (53.33 mW exact)

$\to$ Calculate $I_{in,M} = \frac{V_g}{Z_0 + Z_0} = \frac{16}{600} = 0.02\bar{6}\ A\angle 0°$

$P_{in,m} = \frac{1}{2}|I_{in,m}|^2 Z_0 = \frac{1}{2}(0.02\bar{6})^2\, 300 = \underline{106.667\ mW}$

$\to$ Plot $y_L = 1 - j2\ S/S$ (inductive susceptance) move to

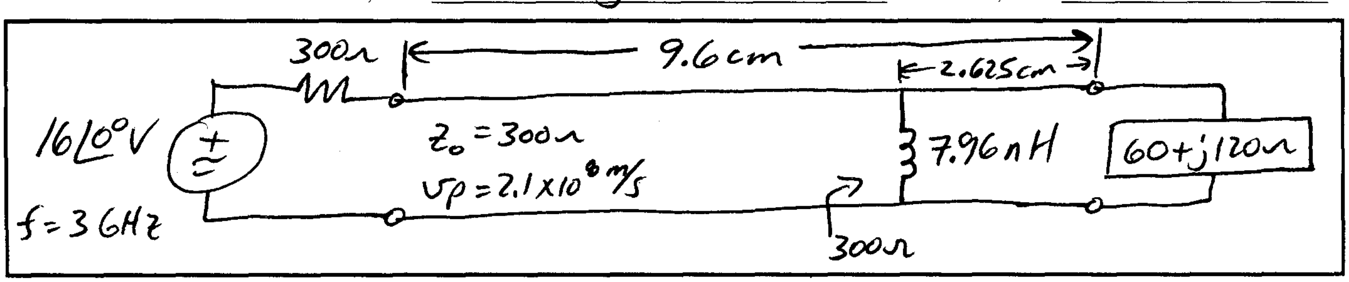
$y_{m1} = 1 + j2\ S/S$ a distance of $d_1 = 0.188\lambda + 0.187\lambda = \underline{0.375\lambda}$

$= 0.375(7) = \underline{2.625\ cm}$

$\to$ Need $y_{ind} = -j2\ S/S \Rightarrow Y_{ind} = -j\frac{2}{300} = -j0.00\bar{6}\ S = -j\frac{1}{\omega L}$

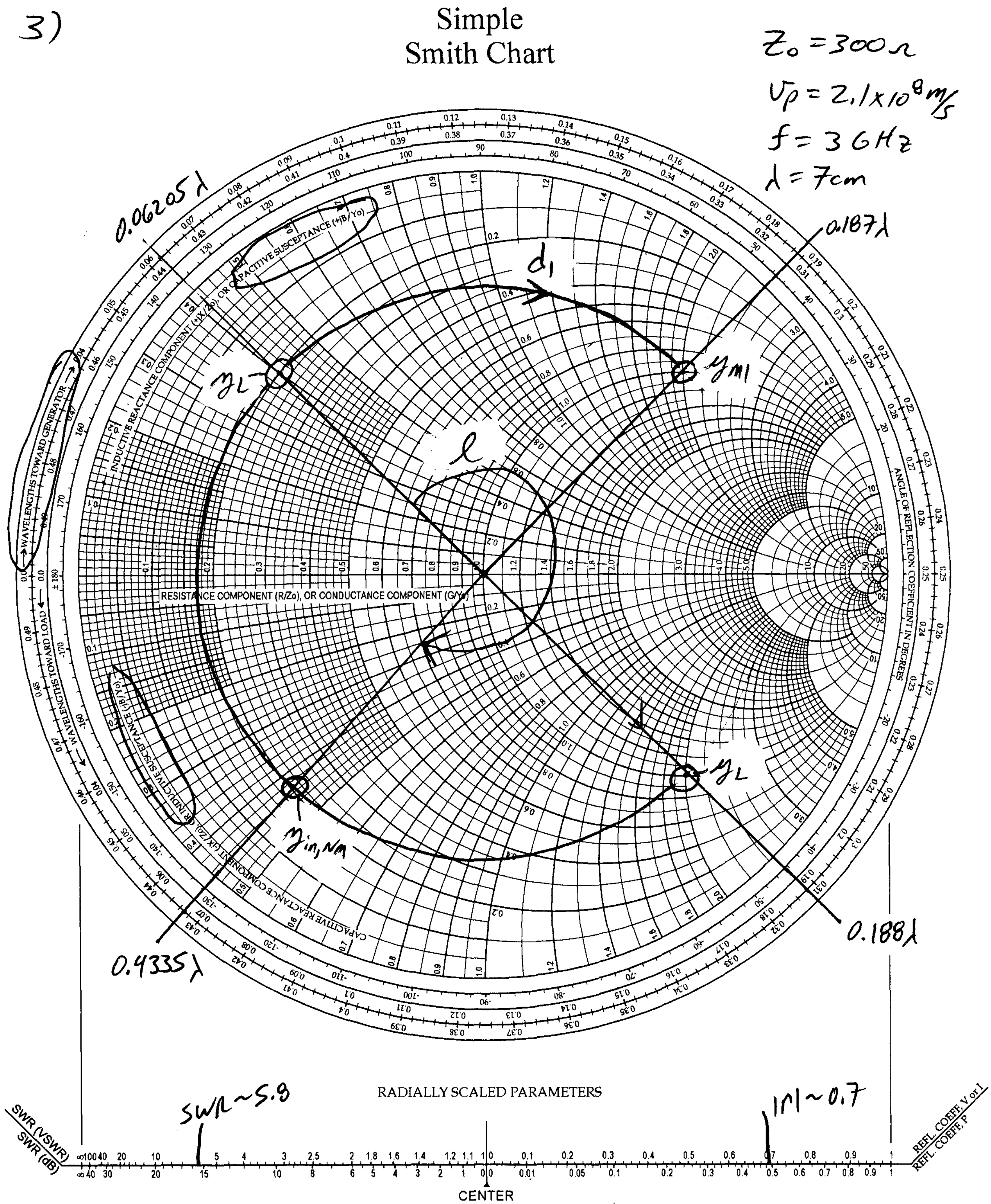
$L = \frac{1}{\omega(0.00\bar{6})} = \frac{1}{2\pi(3\times10^9)(0.00\bar{6})} = \underline{7.9577 \times 10^{-9}\ H}$

$Z_{in,NM}$ = $61.1 - j127.6\ \Omega$ $\qquad P_{L,NM}$ = 53.33 mW



$Z_{in,M}$ = $300\ \Omega$ $\qquad P_{L,M}$ = 106.67 mW

3)


Simple
Smith Chart
$Z_0 = 300\,\Omega$
$v_p = 2.1\times10^{8}$ m/s
$f = 3$ GHz
$\lambda = 7$cm
0.06205λ
0.187λ
d_1
y_L
y_{m1}
l
y_L
$y_{in,NM}$
0.4335λ
0.188λ
RADIALLY SCALED PARAMETERS
SWR~5.8
|Γ|~0.7
CENTER

4) A lossless 40 Ω microstrip TL ($v_p = 2.25 \times 10^8$ m/s) is terminated by a load with reflection coefficient $0.6\angle 114°$ Ω at 1.5 GHz. Find the load impedance and SWR as well as the maximum and minimum impedances along the <u>unmatched</u> TL. Using a Smith chart, design a quarter-wave transformer (QWT) to match the load to the TL. Place the QWT as close as possible to the load. Assume the QWT section has a phase velocity of 2.3×10^8 m/s. Sketch the fully labeled result in box provided.

→ Plot $\Gamma_L = 0.6$ circle on Smith Cht

→ Draw radial line @ 114°, Read $\underline{z_L = 0.345 + j0.59\ \Omega/\Omega}$

$Z_L = z_L Z_0 = (0.345 + j0.59)40 = \underline{13.8 + j23.6\Omega}$

(exact $13.852 + j23.727\Omega$)

→ Calculate $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.6}{1-0.6} = \underline{\underline{4}}$ or use Smith Cht

→ Read $r_{max} = 4$ and $r_{min} = 0.25$

→ Calculate $Z_{max} = r_{max} Z_0 = 4(40) = \underline{\underline{160\Omega}}$

$Z_{min} = r_{min} Z_0 = 0.25(40) = 10\Omega$

→ Per (5.25), $Z_1 = \sqrt{Z_0 Z_L} = \sqrt{40(160)} = \underline{80\Omega}$

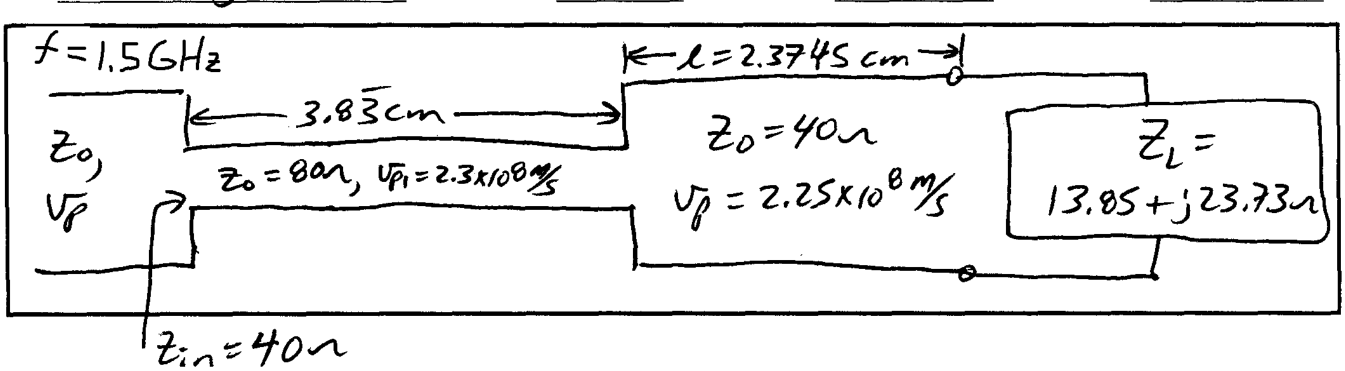
→ $\lambda_1 = \frac{v_{p1}}{f} = \frac{2.3 \times 10^8}{1.5 \times 10^9} = 15.\overline{33}\text{cm} \Rightarrow \underline{\frac{\lambda_1}{4} = 3.8\overline{3}\text{cm}}$

→ $\lambda = \frac{v_p}{f} = \frac{2.25 \times 10^8}{1.5 \times 10^9} = 15\text{cm}$

→ From Smith Chart, the distance from z_L to r_{max} is $\ell = 0.25\lambda = 0.0917\lambda = 0.1583\lambda$

$= 0.1583(15\text{cm}) = \underline{2.3745\text{ cm}}$

Z_L = $13.85 + j23.73\Omega$ SWR = 4 Z_{max} = 160Ω Z_{min} = 10Ω



4)

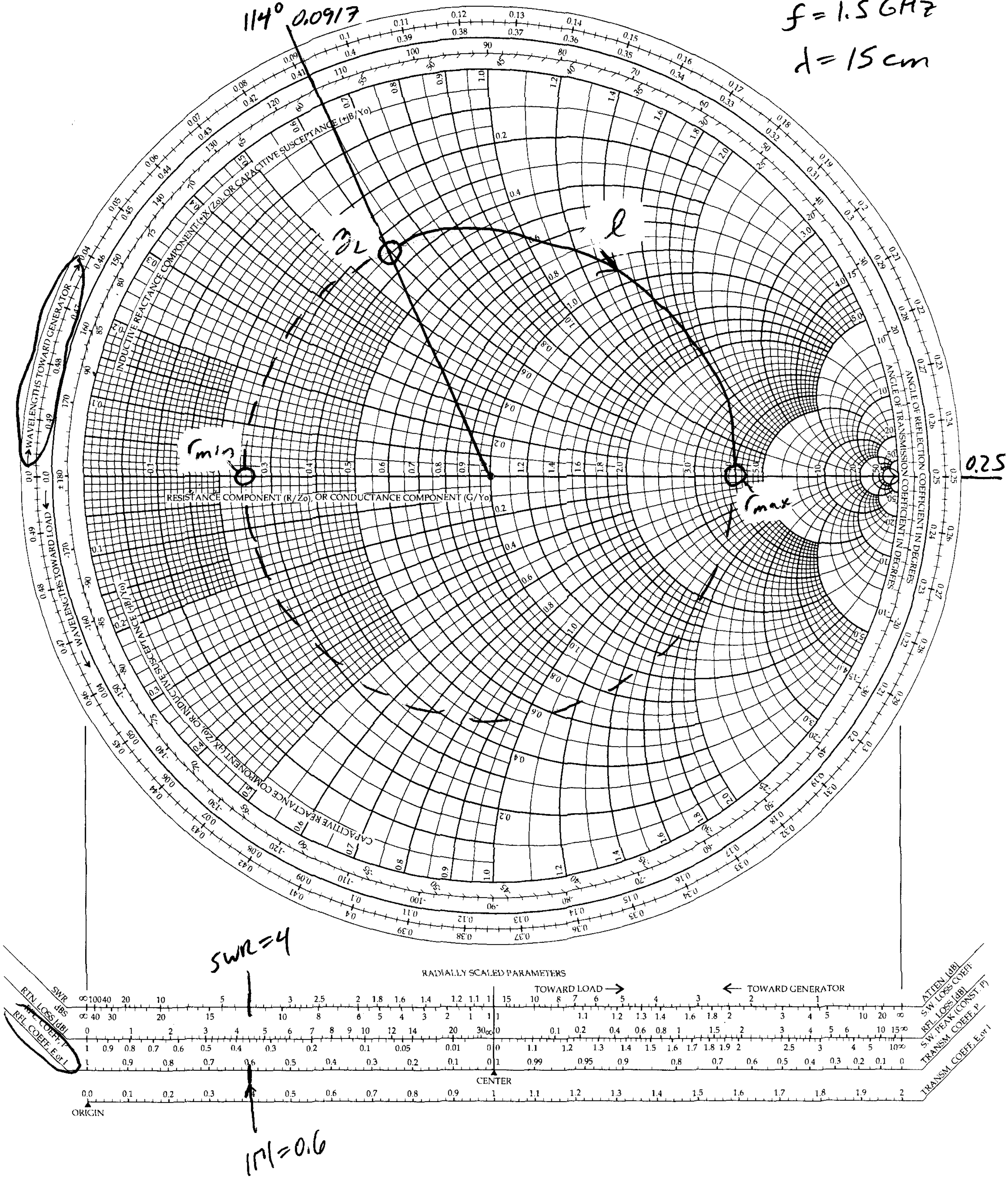
Smith Chart

$Z_0 = 40\Omega$

$v_p = 2.25 \times 10^8$ m/s

$f = 1.5$ GHz

$\lambda = 15$ cm



EE 481/581 Microwave Engineering Examination #1R (Fall 2024)

Name __**KEY R**____

Instructions: Place answers in indicated spaces, use notation as given in class, and show/explain all work for credit. Smith chart should be clearly & fully labeled. Turn-in equation sheet(s) with this work.

1) Problem to replace (i.e, 1, 2, 3, or 4). ____________

2) (20 points) A generator ($V_g = 8\angle 0°$ V, $Z_g = 50\ \Omega$) operating at 600 MHz drives a lossless TL (50 Ω, 2.4×10^8 m/s) of length 130.8 cm connected to a load with a reflection coefficient $0.62\angle 97.1°$. **Using a Smith chart**, find the load impedance Z_L and the unmatched input impedance $Z_{\text{in,NM}}$, reflection coefficient $\Gamma_{\text{in,NM}}$, and SWR. Also, calculate the time-average power delivered to the load $P_{L,\text{NM}}$ without matching.

- Wavelength $\lambda = v_p/f = 2.4\times10^8/600\times10^6 \Rightarrow$ **λ = 0.4 m = 40 cm**.

Smith chart steps

1) Plot $\Gamma_L = 0.62\angle 97.1°$ on Smith chart.

2) Read $z_L = 0.4 + j\ 0.8\ \Omega/\Omega$. Compute $Z_L = Z_0\, z_L = 50\ (0.4 + j\ 0.8) \Rightarrow$ **$Z_L = 20 + j\ 40\ \Omega$**.

3) Draw circle, centered on Smith chart, through Γ_L/z_L point and arc on SWR scale. Read SWR = 4.25.

4) Calculate ℓ/λ = 130.8/40 = 3.27 $\Rightarrow$ move 0.27λ toward generator on arc of constant $|\Gamma|$ to arrive at $z_{\text{in,NM}}$ and $\Gamma_{\text{in,NM}}$ point. Since 0.115+0.27=0.385, draw radial line from center of Smith chart to 0.385 on the "WAVELENGTHS TOWARD GENERATOR" scale.

5) Read $z_{\text{in,NM}} = 0.4 - j\ 0.8\ \Omega/\Omega$ and **$\Gamma_{\text{in,NM}} = 0.62\angle{-97.3°}$**. Compute $Z_{\text{in,NM}} = Z_0\, z_{\text{in,NM}}$ = 50 (0.4 - j 0.8) $\Rightarrow$ **$Z_{\text{in,NM}} = 20 - j\ 40\ \Omega$**.

Use circuit analysis to compute $P_{\text{in,NM}} = P_{L,\text{NM}}$ (lossless TL).

$I_{0,\text{NM}} = V_g/(Z_g + Z_{\text{in,NM}}) = 8\angle 0°/(50 + 20 - j\ 40) = 0.099227788\angle 29.745°$ A

$P_{\text{in,NM}} = 0.5\,|I_{0,\text{NM}}|^2 R_{\text{in,NM}} = 0.5\,|0.099228|^2\, 20$

$\Rightarrow$ **$P_{\text{in,NM}} = P_{L,\text{NM}}$ = 0.0984615 W = 98.4615 mW**.

Not matched

Z_L = __**20 + j 40 Ω**__ $Z_{\text{in,NM}}$ = __**20 - j 40 Ω**__

$\Gamma_{\text{in,NM}}$ = __**0.62∠-97.3°**__ SWR = __4.25__ $P_{L,\text{NM}}$ = __**0.0984615 W = 98.4615 mW**__

Design and sketch (fully labeled) a matching network using a **shunt single capacitor** C placed at a distance d (cm) as close as possible to the load. As part of the process, determine the normalized load admittance y_L and appropriate match point admittance y_M. Find the power delivered to the load after matching $P_{L,M}$.

6) Move 180° around circle of constant $|\Gamma|$ to arrive at y_L = **0.5 - j1 S/S**.

7) The two points are $y_{m,i} = 1 \pm j1.58$ S/S. Choose **$y_{m,2}$ = 1 - j1.58 S/S** (inductive y) to match with a shunt capacitor.

8) The distance in "WAVELENGTHS TOWARD GENERATOR" from y_L to $y_{m,2}$ is d = 0.135 + 0.322 $\Rightarrow$ **$d = 0.457\lambda = 0.457(40) = 18.28$ cm.**

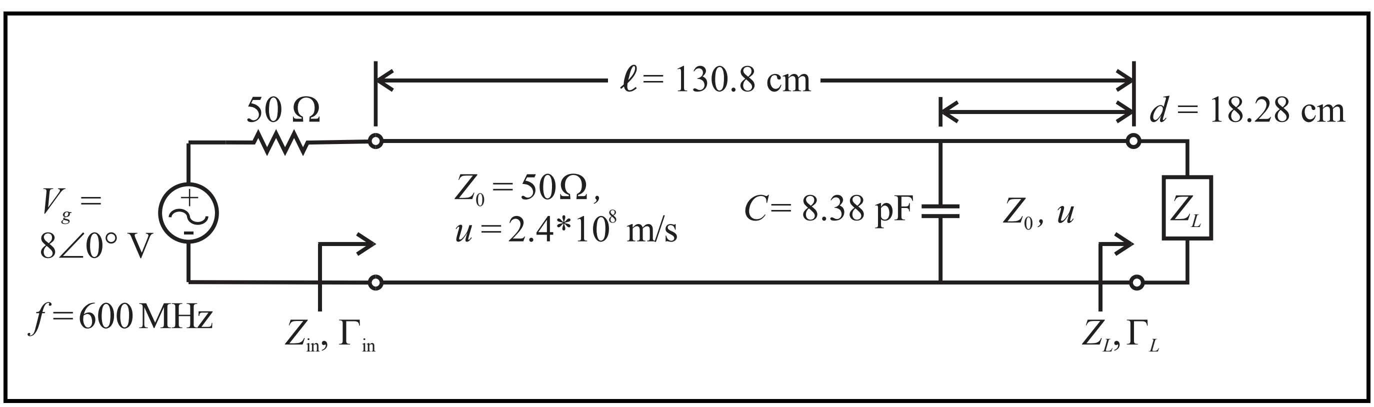
9) At $y_{m,2}$ add a shunt capacitor with susceptance $Y_{cap} = j1.58/50\text{ S} = j\omega C$. Solving for C = $1.58/(2\pi\, 600\times10^6\, 50) \Rightarrow$ **C = 8.38216 pF**.

After matching, $Z_{in,M} = Z_0 = 50\ \Omega$. Use circuit analysis to compute $P_{in,M} = P_{L,M}$ (lossless TL).

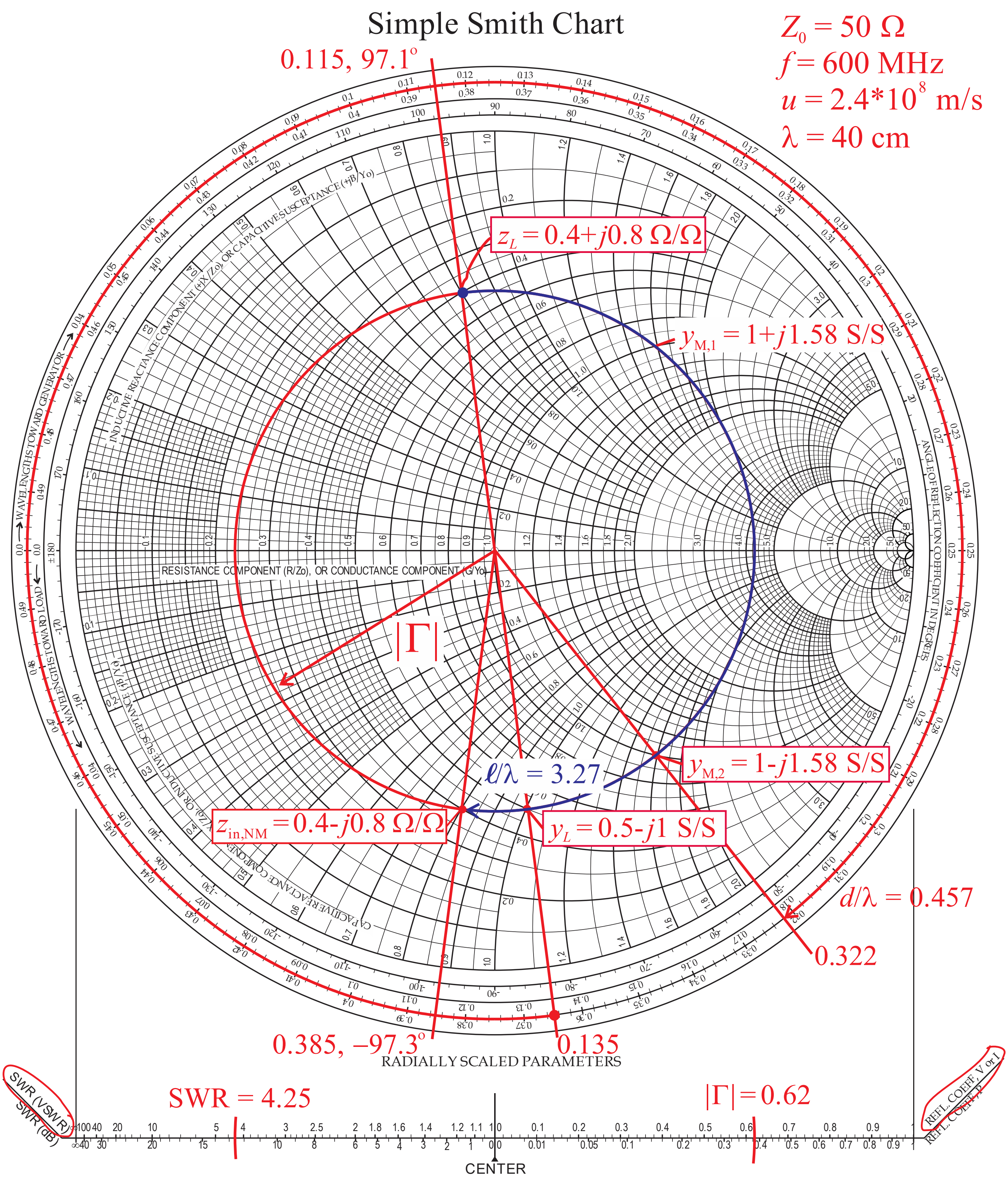
$$I_{0,M} = V_g/(Z_g + Z_{in,M}) = 8\angle0°/(50 + 50) = 0.08\angle0°\text{ A}$$

$$P_{in,M} = 0.5\,|I_{0,M}|^2 R_{in,M} = 0.5\,|0.08|^2\, 50 \quad\Rightarrow\quad \mathbf{P_{in,M} = P_{L,M} = 0.16\text{ W} = 160\text{ mW}}.$$

y_L = **0.5 - j1 S/S** y_M = **1 - j1.58 S/S**



Matched- $P_{L,M}$ = **0.16 W = 160 mW**


Simple Smith Chart
Z0 = 50 Ω
f = 600 MHz
u = 2.4*10^8 m/s
λ = 40 cm
0.115, 97.1°
zL = 0.4+j0.8 Ω/Ω
yM,1 = 1+j1.58 S/S
|Γ|
yM,2 = 1-j1.58 S/S
ℓ/λ = 3.27
zin,NM = 0.4-j0.8 Ω/Ω
yL = 0.5-j1 S/S
d/λ = 0.457
0.322
0.385, −97.3°
0.135
RADIALLY SCALED PARAMETERS
SWR = 4.25
|Γ| = 0.62
SWR (VSWR)
SWR (dB)
REFL. COEFF. V or I
REFL. COEFF. P
CENTER
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo) OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD