# Maxwell's Equations

## Static fields:

| | Integral Form | Differential Form |
|---|---|---|
| Faraday's Law | $\oint_c \bar{E}\cdot d\bar{l} = 0$ | $\bar{\nabla}\times\bar{E} = 0$ |
| Ampere's Law | $\oint_c \bar{H}\cdot d\bar{l} = \int_s \bar{J}\cdot d\bar{s}$ | $\bar{\nabla}\times\bar{H} = \bar{J}$ |
| Gauss' Law | $\oint_s \bar{D}\cdot d\bar{s} = \int_V \rho_v \, dV$ | $\bar{\nabla}\cdot\bar{D} = \rho_v$ |
| | $\oint_s \bar{B}\cdot d\bar{s} = 0$ | $\bar{\nabla}\cdot\bar{B} = 0$ |

## Time-varying fields:

| | Integral Form | Differential Form |
|---|---|---|
| Faraday's Law | $\oint_c \bar{\mathcal{E}}\cdot d\bar{l} = -\frac{d}{dt}\int_s \bar{\mathcal{B}}\cdot d\bar{s}$ | $\bar{\nabla}\times\bar{\mathcal{E}} = -\frac{\partial\bar{\mathcal{B}}}{\partial t}$ |
| Ampere's Law | $\oint_c \bar{\mathcal{H}}\cdot d\bar{l} = \int_s \bar{\mathcal{J}}\cdot d\bar{s} + \int_s \frac{\partial\bar{\mathcal{D}}}{\partial t}\cdot d\bar{s}$ | $\bar{\nabla}\times\bar{\mathcal{H}} = \bar{\mathcal{J}} + \frac{\partial\bar{\mathcal{D}}}{\partial t}$ |
| Gauss' Law | $\oint_s \bar{\mathcal{D}}\cdot d\bar{s} = \int_V \rho_v \, dV$ | $\bar{\nabla}\cdot\bar{\mathcal{D}} = \rho_v$ |
| | $\oint_s \bar{\mathcal{B}}\cdot d\bar{s} = 0$ | $\bar{\nabla}\cdot\bar{\mathcal{B}} = 0$ |

## Time-varying fields, simple media, & stationary circuits:

| | Integral Form | Differential Form |
|---|---|---|
| Faraday's Law | $\oint_c \bar{\mathcal{E}}\cdot d\bar{l} = -\mu\frac{d}{dt}\int_s \bar{\mathcal{H}}\cdot d\bar{s}$ | $\bar{\nabla}\times\bar{\mathcal{E}} = -\mu\frac{\partial\bar{\mathcal{H}}}{\partial t}$ |
| Ampere's Law | $\oint_c \bar{\mathcal{H}}\cdot d\bar{l} = \sigma\int_s \bar{\mathcal{E}}\cdot d\bar{s} + \varepsilon\frac{d}{dt}\int_s \bar{\mathcal{E}}\cdot d\bar{s} + \int_s \bar{\mathcal{J}}\cdot d\bar{s}$ | $\bar{\nabla}\times\bar{\mathcal{H}} = \sigma\bar{\mathcal{E}} + \varepsilon\frac{\partial\bar{\mathcal{E}}}{\partial t} + \bar{\mathcal{J}}$ |
| Gauss' Law | $\oint_s \bar{\mathcal{E}}\cdot d\bar{s} = \frac{1}{\varepsilon}\int_V \rho_v \, dV$ | $\bar{\nabla}\cdot\bar{\mathcal{E}} = \frac{\rho_v}{\varepsilon}$ |
| | $\oint_s \bar{\mathcal{H}}\cdot d\bar{s} = 0$ | $\bar{\nabla}\cdot\bar{\mathcal{H}} = 0$ |

# Time-harmonic/sinusoidal steady-state time-varying fields:

| | Integral Form | Differential Form |
|---|---|---|
| Faraday's Law | $\oint_c \hat{\bar{E}} \bullet d\bar{l} = -j\omega \int_s \hat{\bar{B}} \bullet d\bar{s}$ | $\bar{\nabla} \times \hat{\bar{E}} = -j\omega \hat{\bar{B}}$ |
| Ampere's Law | $\oint_c \hat{\bar{H}} \bullet d\bar{l} = \int_s \hat{\bar{J}} \bullet d\bar{s} + j\omega \int_s \hat{\bar{D}} \bullet d\bar{s}$ | $\bar{\nabla} \times \hat{\bar{H}} = \hat{\bar{J}} + j\omega \hat{\bar{D}}$ |
| Gauss' Law | $\oint_s \hat{\bar{D}} \bullet d\bar{s} = \int_V \hat{\rho}_v \, dV$ | $\bar{\nabla} \bullet \hat{\bar{D}} = \hat{\rho}_v$ |
| | $\oint_s \hat{\bar{B}} \bullet d\bar{s} = 0$ | $\bar{\nabla} \bullet \hat{\bar{B}} = 0$ |

# Time-harmonic/sinusoidal steady-state time-varying fields & mimple media:

| | Integral Form | Differential Form |
|---|---|---|
| Faraday's Law | $\oint_c \hat{\bar{E}} \bullet d\bar{l} = -j\omega\mu \int_s \hat{\bar{H}} \bullet d\bar{s}$ | $\bar{\nabla} \times \hat{\bar{E}} = -j\omega\mu \hat{\bar{H}}$ |
| Ampere's Law | $\oint_c \hat{\bar{H}} \bullet d\bar{l} = (\sigma + j\omega\varepsilon) \int_s \hat{\bar{E}} \bullet d\bar{s} + \int_s \hat{\bar{J}} \bullet d\bar{s}$ | $\bar{\nabla} \times \hat{\bar{H}} = (\sigma + j\omega\varepsilon) \hat{\bar{E}} + \hat{\bar{J}}$ |
| Gauss' Law | $\oint_s \hat{\bar{E}} \bullet d\bar{s} = \frac{1}{\varepsilon} \int_V \hat{\rho}_v \, dV$ | $\bar{\nabla} \bullet \hat{\bar{E}} = \frac{\hat{\rho}_v}{\varepsilon}$ |
| | $\oint_s \hat{\bar{H}} \bullet d\bar{s} = 0$ | $\bar{\nabla} \bullet \hat{\bar{H}} = 0$ |

# Other important relationships:

| | Integral Form | Differential Form |
|---|---|---|
| Eqn of Continuity / Conservation of Charge | $\oint_s \bar{\mathcal{J}} \bullet d\bar{s} = -\frac{d}{dt}\int_V \rho_v \, dV$ | $\bar{\nabla} \bullet \bar{\mathcal{J}} = -\frac{\partial \rho_v}{\partial t}$ |

Lorentz Force Eqn. $\bar{\mathcal{F}} = q(\bar{\mathcal{E}} + \bar{u} \times \bar{\mathcal{B}})$

Constitutive Relations $\bar{\mathcal{D}} = \varepsilon \bar{\mathcal{E}} = \varepsilon_r \varepsilon_0 \bar{\mathcal{E}}$ $\quad \bar{\mathcal{B}} = \mu \bar{\mathcal{H}} = \mu_r \mu_0 \bar{\mathcal{H}}$

Ohm's Law $\bar{\mathcal{J}}_c = \sigma \bar{\mathcal{E}}$

| Boundary Conditions | | Electric | Magnetic |
|---|---|---|---|
| | Tangential- | $\bar{\mathcal{E}}_{1t} = \bar{\mathcal{E}}_{2t}$ or $\hat{a}_{n12} \times (\bar{\mathcal{E}}_2 - \bar{\mathcal{E}}_1) = 0$ | $\hat{a}_{n12} \times (\bar{\mathcal{H}}_2 - \bar{\mathcal{H}}_1) = \bar{\mathcal{J}}_s$ |
| | Normal- | $\hat{a}_{n12} \bullet (\bar{\mathcal{D}}_2 - \bar{\mathcal{D}}_1) = \rho_s$ | $\bar{\mathcal{B}}_{1n} = \bar{\mathcal{B}}_{2n}$ or $\hat{a}_{n12} \bullet (\bar{\mathcal{B}}_2 - \bar{\mathcal{B}}_1) = 0$ |

where surface normal $\hat{a}_{n12}$ points from region 1 into region 2, and $\mathcal{B}_{1n}$ & $\mathcal{D}_{1n}$ point away from boundry while $\mathcal{B}_{2n}$ & $\mathcal{D}_{2n}$ point toward from boundry

Permittivity of free space, $\varepsilon_0 = 8.8541878 \times 10^{-12}$ F/m

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H/m

Poynting Vector $\bar{\mathcal{S}} = \bar{\mathcal{E}} \times \bar{\mathcal{H}}$

**Poynting Theorem**

Differential Form- $-\bar{\nabla} \bullet \bar{\mathcal{S}} = \bar{\mathcal{E}} \bullet \bar{\mathcal{J}} + \bar{\mathcal{E}} \bullet \frac{\partial \bar{\mathcal{D}}}{\partial t} + \bar{\mathcal{H}} \bullet \frac{\partial \bar{\mathcal{B}}}{\partial t}$

Integral Form- $-\oint_s \bar{\mathcal{S}} \bullet d\bar{s} = \int_V \bar{\mathcal{E}} \bullet \bar{\mathcal{J}} \, dV + \int_V \left( \bar{\mathcal{E}} \bullet \frac{\partial \bar{\mathcal{D}}}{\partial t} \right) dV + \int_V \left( \bar{\mathcal{H}} \bullet \frac{\partial \bar{\mathcal{B}}}{\partial t} \right) dV$