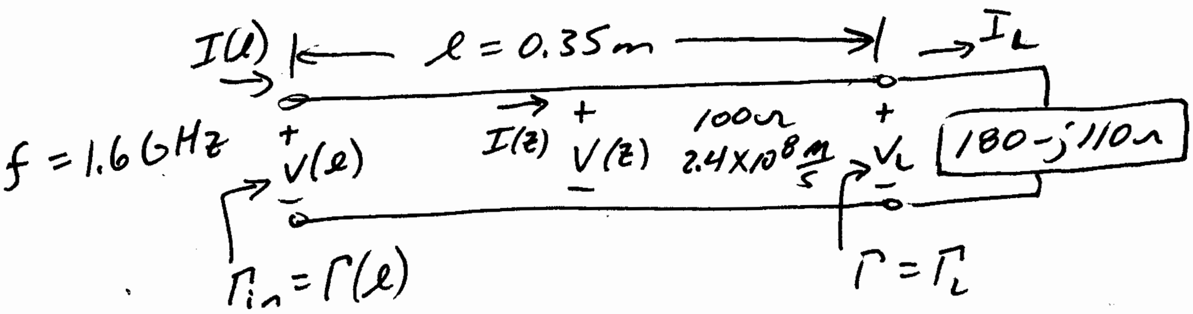
2.3 cont.

ex. Calculate various quantities for a lossless TL ($Z_0 = 100\Omega$, $u_p = 2.4\times10^8$ m/s) of length $\ell = 0.35$ m terminated in a load $Z_L = 180 - j110\Omega$ operating @ 1.6 GHz w/ $V_0^+ = 10\angle 40^\circ$ V @ $z = 0$.



$$\lambda = \frac{v_p}{f} = \frac{2.4\times10^8}{1.6\times10^9} = \underline{0.15\text{ m}}$$

$$\ell/\lambda = \frac{0.35}{0.15} = \underline{2.\overline{33}} \leftarrow \text{TL length in terms of } \lambda$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v_p} = \frac{2\pi(1.6\times10^9)}{2.4\times10^8} = \underline{41.8879\ \tfrac{\text{rad}}{\text{m}}}$$

$$\Gamma = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(180-j110)-100}{(180-j110)+100} = \underline{0.4521\angle -32.525^\circ}$$

$$\Gamma(\ell) = \Gamma_{in} = \Gamma e^{-j2\beta\ell} = (0.4521\angle -32.525^\circ)\, e^{-j2(41.89)0.35}$$

$$\underline{\Gamma(\ell) = \Gamma_{in} = 0.4521\angle 187.475^\circ}$$

$$Z_{in} = Z_0\left(\frac{1+\Gamma_{in}}{1-\Gamma_{in}}\right) = 100\left(\frac{1+0.452\angle 87.5^\circ}{1-0.452\angle 87.5^\circ}\right) = \underline{68.3145 + j77.57\Omega}$$

$$RL = -20\log_{10} 0.4521 = \underline{6.895\text{ dB}}$$

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.4521}{1-0.4521} = \underline{2.6505}$$

} Not a good match

ex. cont. Find various voltage-related items

$$V_o^- = V_o^+ \Gamma = (10\angle 40^\circ)(0.4521\angle -32.525^\circ) = \underline{4.5213\angle 7.475^\circ\, V}$$

$$V_L = V_o^+ \underbrace{e^{-j\beta(0)}}_{1} + V_o^- \underbrace{e^{+j\beta(0)}}_{1} = (10\angle 40^\circ) + (4.5213\angle 7.475^\circ)$$

$$\underline{V_L = 14.0244\angle 30.018^\circ\, V}$$

$$V_{max} = |V_o^+|(1+|\Gamma|) = |10\angle 40^\circ|(1+0.4521) = \underline{14.521\, V}$$

$$V_{min} = |V_o^+|(1-|\Gamma|) = 10(1-0.4521) = \underline{5.479 V}$$

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

$$\underline{V(z) = (10\angle 40^\circ)\, e^{-j41.89z} + (4.52\angle 7.5^\circ) e^{+j41.89z} \quad V}$$

$$\underline{\text{for } -0.35\text{m} \le z \le 0}$$

Find various current-related items

$$I_o^+ = \frac{V_o^+}{Z_o} = \frac{10\angle 40^\circ}{100} = \underline{0.1\angle 40^\circ A}$$

$$I_o^- = -\frac{V_o^-}{Z_o} = \frac{-(4.52\angle 7.5^\circ)}{100} = \underline{0.0452\angle -172.525^\circ A}$$

$$I_L = I_o^+ e^0 + I_o^- e^0 = (0.1\angle 40^\circ) + (0.0452\angle -172.525^\circ)$$

$$\underline{I_L = 0.0665\angle 61.448^\circ A} \quad \left(\text{or } I_L = V_L/Z_L\right)$$

$$I_{max} = \frac{|V_o^+|}{Z_o}(1+|\Gamma|) = |I_o^+|(1+|\Gamma|) = 0.1(1+0.452) = \underline{0.1452 A}$$

$$I_{min} = |I_o^-|(1-|\Gamma|) = 0.0452(1-0.452) = \underline{0.0548 A}$$

$$\underline{I(z) = I_o^+ e^{-j\beta z} + I_o^- e^{j\beta z} = (0.1\angle 40^\circ) e^{-j41.89z} + (0.0452\angle -172.5^\circ) e^{j41.89z} \; A}$$

$$-0.35\text{m} \le z \le 0$$

ex. cont. Find some power-related quantities

$$P_{ave,L} = \tfrac{1}{2}\,\mathrm{Re}\{V_L I_L^*\} = 0.5\,\mathrm{Re}\{(14.02\angle 30^\circ)(0.0665\angle -61.45^\circ)\}$$

$$\underline{P_{ave,L} = 0.3978\ \mathrm{W}}$$

$$P_{avg} = \frac{1}{2}\frac{|V_o^+|^2}{Z_o}\left(1-|\Gamma|^2\right) = \frac{1}{2}\frac{10^2}{100}\left(1-0.4521^2\right)$$

$\underline{P_{avg} = 0.3978\ \mathrm{W}}$ same!

$$P_{avg,inc} = P_{avg}^+ = \tfrac{1}{2}\frac{|V_o^+|^2}{Z_o} = \frac{1}{2}\frac{10^2}{100} = \underline{0.5\ \mathrm{W}}$$

$$P_{avg,ref} = P_{avg}^- = \frac{1}{2}\frac{|V_o^+|^2}{Z_o}|\Gamma|^2 = \frac{1}{2}\frac{10^2}{100}0.4521^2 = \underline{0.1022\ \mathrm{W}}$$

see following MathCad pages for confirmation of numbers/calculations as well as plots of $|V(z)|$, $|I(z)|$, and $P_{ave}(z)$ for $-0.35\,\mathrm{m} \le z \le 0$.

Find location of V_{max} (+ I_{min}) closest to load

For V_{max} (+ I_{min}), $\Gamma e^{j2\beta z} = |\Gamma|(1) \Rightarrow 1\angle -32.525^\circ\, e^{j2\beta z} = 1$

$$e^{j(-32.525\,\pi/180 + 2(41.888)z)} = e^{j0}$$

$$\hookrightarrow -32.525\,\pi/180 + 2(41.888)z = 0$$

$$z = 0.006776\ \mathrm{m} \leftarrow \text{NOT possible}$$

$$z = 0.006776 - \lambda/2 = 0.006776 - \frac{0.15}{2}$$

$$\underline{\underline{z = -0.068224\ \mathrm{m}}}$$

Enter given information

$V0p := 10 \cdot e^{j \cdot 40 \cdot \frac{\pi}{180}}$ V $\quad f := 1.6 \cdot 10^{9}$ Hz $\quad ZL := 180 - j \cdot 110$ Ω

$l := 0.35$ m $\quad vp := 2.4 \cdot 10^{8}$ m/s $\quad Z0 := 100$ Ω

Calculate variables related to transmission line

$\omega := 2 \cdot \pi \cdot f \quad \lambda := \frac{vp}{f} \quad \boxed{\lambda = 0.15}$ m $\quad l\lambda := \frac{l}{\lambda} \quad \boxed{l\lambda = 2.333}$

$\beta := \frac{\omega}{vp} \quad \boxed{\beta = 41.8879}$ rad/m $\quad n := 0 .. 466 \quad z_n := \frac{-n}{466} \cdot l$

Calculate reflection coefficients, return loss, SWR, & input impedance

$\Gamma := \frac{ZL - Z0}{ZL + Z0} \quad \boxed{|\Gamma| = 0.4521} \quad \boxed{\arg(\Gamma) \cdot \frac{180}{\pi} = -32.525}$ deg

$RL := -20 \cdot \log(|\Gamma|) \quad \boxed{RL = 6.895}$ dB

$\Gamma l := \Gamma \cdot e^{-j \cdot 2 \cdot \beta \cdot l} \quad \boxed{|\Gamma l| = 0.4521} \quad \boxed{\arg(\Gamma l) \cdot \frac{180}{\pi} = 87.475}$ deg

$Zin := Z0 \cdot \frac{(1 + \Gamma l)}{(1 - \Gamma l)} \quad \boxed{Zin = 68.3145 + 77.5709i}$ Ω

$SWR := \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \boxed{SWR = 2.6505}$

Calculate V0m, VL, Vmax, Vmin, & phasor voltage

$V0m := V0p \cdot \Gamma \quad \boxed{|V0m| = 4.5213}$ V $\quad \boxed{\arg(V0m) \cdot \frac{180}{\pi} = 7.475}$ deg

$VL := V0p + V0m \quad \boxed{|VL| = 14.0244}$ V $\quad \boxed{\arg(VL) \cdot \frac{180}{\pi} = 30.018}$ deg

$V_n := V0p \cdot e^{-j \cdot \beta \cdot z_n} + V0m \cdot e^{j \cdot \beta \cdot z_n}$ Phasor voltage versus position along TL.

$Vmax := |V0p| \cdot (1 + |\Gamma|) \quad \boxed{Vmax = 14.5213}$ V

$Vmin := |V0p| \cdot (1 - |\Gamma|) \quad \boxed{Vmin = 5.4787}$ V $\quad \frac{Vmax}{Vmin} = 2.6505$

Calculate I0p, I0m, IL, Imax, Imin, & phasor current

$I0p := \frac{V0p}{Z0}$ $\boxed{|I0p| = 0.1}$ A $\boxed{\arg(I0p) \cdot \frac{180}{\pi} = 40}$ deg

$I0m := \frac{-V0m}{Z0}$ $\boxed{|I0m| = 0.0452}$ A $\boxed{\arg(I0m) \cdot \frac{180}{\pi} = -172.525}$ deg

$IL := I0p + I0m$ $\boxed{|IL| = 0.0665}$ A $\boxed{\arg(IL) \cdot \frac{180}{\pi} = 61.448}$ deg

$ILalt := \frac{VL}{ZL}$ $\boxed{|ILalt| = 0.0665}$ A $\boxed{\arg(ILalt) \cdot \frac{180}{\pi} = 61.448}$ deg

$I_n := \frac{V0p}{Z0} \cdot e^{-j \cdot \beta \cdot z_n} - \frac{V0m}{Z0} \cdot e^{j \cdot \beta \cdot z_n}$ Phasor current versus position along TL.

$Imax := \frac{|V0p|}{Z0} \cdot (1 + |\Gamma|)$ $\boxed{Imax = 0.1452}$ A

$Imin := \frac{|V0p|}{Z0} \cdot (1 - |\Gamma|)$ $\boxed{Imin = 0.0548}$ A

Calculate time-average total, incident, & reflected powers

$Pavg1 := 0.5 \cdot \mathrm{Re}\left(VL \cdot \overline{IL}\right)$ $\boxed{Pavg1 = 0.3978}$ W

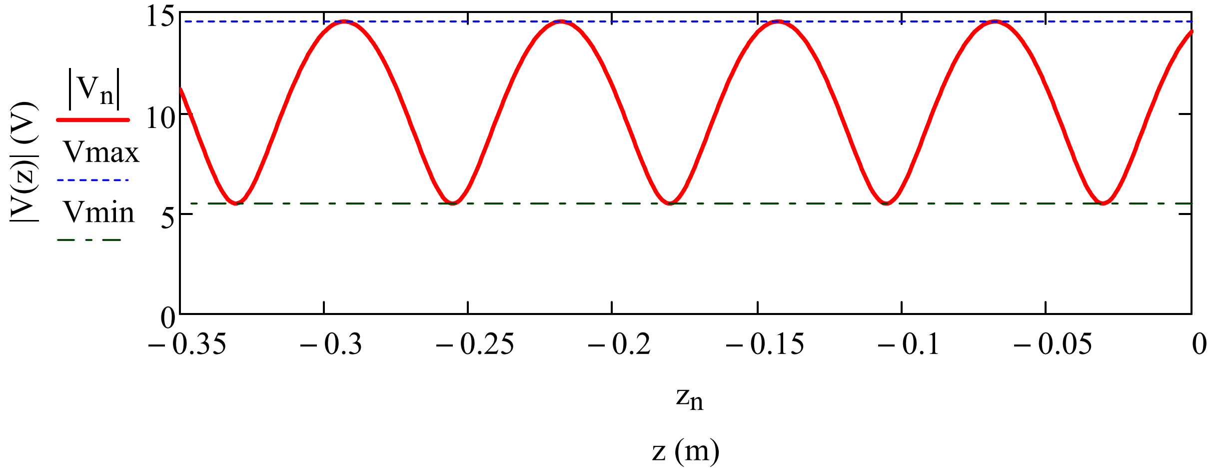
$Pavg := 0.5 \cdot \frac{(|V0p|)^2}{Z0} \cdot \left[1 - (|\Gamma|)^2\right]$ $\boxed{Pavg = 0.3978}$ W

$Pavg\_inc := 0.5 \cdot \frac{(|V0p|)^2}{Z0}$ $\boxed{Pavg\_inc = 0.5}$ W

$Pavg\_ref := 0.5 \cdot \frac{(|V0p|)^2}{Z0} \cdot (|\Gamma|)^2$ $\boxed{Pavg\_ref = 0.1022}$ W

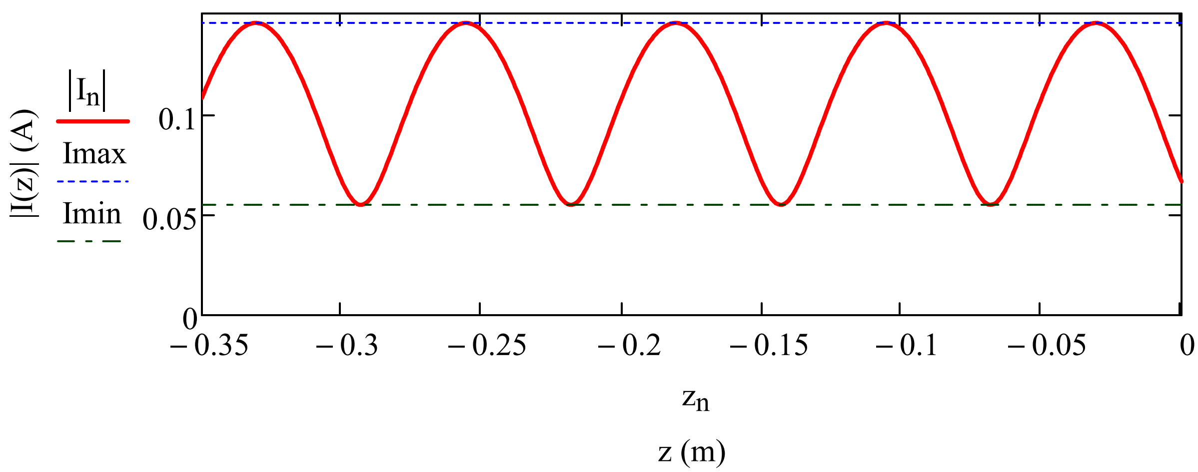
$Pave_n := 0.5 \cdot \mathrm{Re}\left(V_n \cdot \overline{I_n}\right)$ Check to see if power is really constant.

Phasor voltage magnitude vs position



$|V_{91}| = 14.521$ V $\quad z_{91} = -0.0683$ m $\quad |V_{41}| = 5.479$ V $\quad z_{41} = -0.0308$ m

Phasor current magnitude vs position



$|I_{41}| = 0.145$ A $\quad z_{41} = -0.0308$ m $\quad |I_{91}| = 0.0548$ A $\quad z_{91} = -0.0683$ m

Time-ave Power vs position