



For the lossless transmission line circuit shown:  $f = 100 \text{ MHz}$ ,  $v_p = 3 \times 10^8 \text{ m/s}$ ,  $l = 3.3 \text{ m}$ ,  $Z_0 = 50 \Omega$ , and  $Z_L = 75 + j50 \Omega$ .

### 1) Normalize and plot load impedance

- Normalize  $z_L = Z_L / Z_0 = (75 + j50) / 50 \Rightarrow \underline{z_L = 1.5 + j1 \Omega/\Omega}$ .
- Plot  $z_L$  on Smith chart by finding intersection of  $r = 1.5$  circle with  $x = 1$  arc.

### 2) Find load reflection coefficient and VSWR

- Set compass to distance between center of Smith chart and  $z_L$ . Use compass to mark the “REFL. COEFF. V or I” scale at bottom right of Smith chart to determine  $|\Gamma_L| = 0.42$ .
- Use compass to draw  $|\Gamma| = 0.42$  arc, centered on Smith chart scales, through SWR (VSWR) scale on bottom left. Read **VSWR = 2.4**.
- Use straight-edge to draw radial line from center of Smith chart through  $z_L$  and outer rings of Smith chart. Use “ANGLE OF REFLECTION COEFFICIENT IN DEGREES” scale to read  $\angle \Gamma_L = 41.8^\circ$ .
- Put magnitude and angle together to get  **$\Gamma_L = 0.42 \angle 41.8^\circ$** . For comparison, the analytic result is  $\Gamma_L = 0.4152 \angle 41.63^\circ$ .

### 3) Find input reflection coefficient

- Calculate  $l/\lambda = lf/v_p = 3.3(100 \times 10^6) / 3 \times 10^8 = 1.1$ . Subtract  $2(0.5) = 1$  (i.e., remove integer multiples of  $n\lambda/2$ ) to get  $\Rightarrow \underline{l/\lambda = 0.1}$ .
- On the Smith chart, the radial line through  $z_L$  reads 0.192 on the “WAVELENGTHS TOWARD GENERATOR” scale. Add  $0.192 + l/\lambda$  to get 0.292 and draw a radial line from the center of the Smith chart through this point on the scale.

- Draw an arc, centered on Smith chart, from  $z_L$  through radial line at 0.292. The intersection of the arc and radial line is the  $\Gamma_{in} / z_{in}$  point. Use the “ANGLE OF REFLECTION COEFFICIENT IN DEGREES” scale to read  $\angle \Gamma_{in} = -30.2^\circ$  and note  $|\Gamma_{in}| = |\Gamma_L| = 0.42$ .
- Put magnitude and angle together to get  $\Gamma_{in} = \mathbf{0.42 \angle -30.2^\circ}$ .

#### 4) Find input impedance

- At  $\Gamma_{in} = 0.42 \angle -30.2^\circ$  point, locate and read/interpolate value of appropriate “ $r$ ” circle as  $r_{in} = 1.8$ .
- At  $\Gamma_{in} = 0.42 \angle -30.2^\circ$  point, locate and read/interpolate value of appropriate “ $x$ ” arc as  $x_{in} = -0.92$ .
- Put together to get normalized input impedance  $z_{in} = \mathbf{1.8 - j0.92 \Omega/\Omega}$ .
- Find input impedance by multiplying  $z_{in}$  w/ characteristic impedance to get  $Z_{in} = Z_0 z_{in} = 50(1.8 - j0.92) \Rightarrow \mathbf{Z_{in} = 90 - j46 \Omega}$ .

Simple  
Smith Chart

$Z_0 = 50 \Omega$   
 $f = 100 \text{ MHz}$   
 $V_p = 3 \times 10^8 \text{ m/s}$

