EE382 Applied Electromagnetics Quiz #1 (Spring 2018)

Name Key A

Instructions: Open book/notes. Show all work for credit.

The phasor vector electric field $\overline{E} = \hat{a}_\phi\, j15\sin\theta \dfrac{e^{-jkr}}{4\pi r}$ (V/m), where $k = \omega/c = 2\pi/\lambda$, exists in free space.

First, find the **time-domain (T-D)** electric field if the frequency is 3.1831 MHz with all constants evaluated (e.g., k & ω). Next, calculate the **time-domain** displacement current density and evaluate at the spherical point A(1.9 m, $2\pi/5$, $3\pi/5$) at time $t = 2$ ms.

$$K = \frac{\omega}{c} = \frac{2\pi(3.1831\times10^6)}{2.99792458\times10^8} = 0.0667128 \ \frac{rad}{m}$$

$$\omega = 2\pi(3.1831\times10^6) = 20\times10^6 \ rad/s$$

$$\overline{E} = \hat{a}_\phi\, j\, 15\sin\theta \frac{e^{-jkr}}{4\pi r} = \hat{a}_\phi \frac{15\sin\theta}{4\pi r} e^{j\pi/2} e^{-jkr} \qquad j = e^{j\pi/2}$$

$$= \hat{a}_\phi \frac{1.19366}{r} \sin\theta\, e^{j(-kr+\pi/2)}$$

$$\overline{\mathcal{E}} = \mathcal{R}e\{\overline{E}e^{j\omega t}\} = \mathcal{R}e\left\{\hat{a}_\phi \frac{1.19366}{r}\sin\theta\, e^{j(\omega t - kr + \pi/2)}\right\}$$

$$= \hat{a}_\phi \frac{1.19366}{r}\sin\theta\cos(20\times10^6 t - 0.066713 r + \pi/2) \ V/m$$

$$(9.22b)\ \overline{\mathcal{J}}_d = \frac{\partial \overline{\mathcal{D}}}{\partial t} = \frac{\partial(\epsilon_0 \overline{\mathcal{E}})}{\partial t} = \frac{\partial}{\partial t}\left[\hat{a}_\phi \frac{15\epsilon_0\sin\theta}{4\pi r}\cos(\omega t - kr + \pi/2)\right]$$

$$= \hat{a}_\phi \frac{15\epsilon_0 \sin\theta}{4\pi r}\,\omega\left(-\sin(\omega t - kr + \pi/2)\right)$$

$$= -\hat{a}_\phi \frac{15(8.854\times10^{-12})(20\times10^6)}{4\pi r}\sin\theta\, \underbrace{\sin(\omega t - kr + \pi/2)}_{\cos(\omega t - kr)}$$

$$= -\hat{a}_\phi\, 2.11378\times10^{-4}\frac{\sin\theta}{r}\cos(20\times10^6 t - 0.00667 r) \ A/m^2$$

T-D electric field = $\hat{a}_\phi \frac{1.19366}{r}\sin\theta\cos(20\times10^6 t - 0.00667 r + \pi/2)$ V/m

T-D displacement current density = $-\hat{a}_\phi\, 2.11378\times10^{-4}\frac{\sin\theta}{r}\cos(20\times10^6 t - 0.00667 r)$ A/m²

T-D displacement current density @ A w/ $t = 2$ ms = $-\hat{a}_\phi\, 4.5155\times10^{-5}$ A/m²

$= -\hat{a}_\phi\, 45.155\ \mu A/m^2$

EE382 Applied Electromagnetics Quiz #1 (Spring 2018)

Name Key B

Instructions: Open book/notes. Show all work for credit.

The phasor vector electric field $\bar{E} = \hat{a}_\phi\, j25\sin\theta \dfrac{e^{-jkr}}{4\pi r}$ (V/m), where $k = \omega/c = 2\pi/\lambda$, exists in free space. First, find the **time-domain (T-D)** electric field if the frequency is 6.3662 MHz with all constants evaluated (e.g., k & ω). Next, calculate the **time-domain** displacement current density and evaluate at the spherical point B(2.3 m, 3π/5, 2π/5) at time t = 3 ms.

$$K = \omega/c = \frac{2\pi(6.3662\times10^6)}{2.99792458\times10^8} = 0.133425686 \ \frac{rad}{m}$$

$$\omega = 2\pi(6.3662\times10^6) = 40\times10^6 \ rad/s$$

$$\bar{E} = \hat{a}_\phi\, j25\sin\theta \frac{e^{-jkr}}{4\pi r} = \hat{a}_\phi \frac{25\sin\theta}{4\pi r} e^{j\pi/2} e^{-jkr} \qquad j = e^{+j\pi/2}$$

$$= \hat{a}_\phi \frac{1.98944}{r} \sin\theta\, e^{j(\pi/2 - kr)}$$

$$\bar{\mathcal{E}} = \mathcal{Re}\{\bar{E} e^{j\omega t}\} = \mathcal{Re}\left\{\hat{a}_\phi \frac{1.98944}{r}\sin\theta\, e^{j(\omega t - kr + \pi/2)}\right\}$$

$$= \hat{a}_\phi \frac{1.98944}{r}\sin\theta\cos(\omega t - kr + \pi/2) \ V/m$$

$$(9.22b)\quad \bar{\mathcal{J}}_d = \frac{\partial \bar{\mathcal{D}}}{\partial t} = \frac{\partial(\epsilon_0 \bar{\mathcal{E}})}{\partial t} = \frac{\partial}{\partial t}\left[\hat{a}_\phi \frac{25\epsilon_0 \sin\theta}{4\pi r}\cos(\omega t - kr + \pi/2)\right]$$

$$= \hat{a}_\phi \frac{25\epsilon_0\sin\theta}{4\pi r}\omega\left(-\sin(\omega t - kr + \pi/2)\right)$$

$$= \hat{a}_\phi \frac{-25(8.8542\times10^{-12})(40\times10^6)}{4\pi}\frac{\sin\theta}{r}\sin(\omega t - kr + \pi/2)$$

$$= -\hat{a}_\phi\, 7.04594\times10^{-4}\frac{\sin\theta}{r}\underbrace{\sin(\omega t - kr + \pi/2)}_{\cos(\omega t - kr)} \qquad \text{Trig ID. } \sin(A + \pi/2) = \cos A$$

$$\bar{\mathcal{J}}_{d,B}(3ms) = -\hat{a}_\phi\, 7.046\times10^{-4}\frac{\sin 3\pi/5}{2.3}\cos(40\times10^6(0.003) - 0.13343(2.3))$$

T-D electric field = $\hat{a}_\phi \dfrac{1.98944}{r}\sin\theta\cos(40\times10^6 t - 0.13343 r + \pi/2)$ V/m

T-D displacement current density = $-\hat{a}_\phi\, 7.046\times10^{-4}\dfrac{\sin\theta}{r}\cos(40\times10^6 t - 0.13343 r)$ A/m²

T-D displacement current density @ B w/ t = 3 ms = $\hat{a}_\phi\, 2.7643\times10^{-4}$ A/m²

$= \hat{a}_\phi\, 276.43\ \mu A/m^2$