## EE382 Homework \#3

Assigned: Mon., Jan. 22, 2018
Due: Fri, Jan. 26, 2018
Total Points: 40
(1) A rectangular loop rotates about the $z$ axis in the figure below an angular speed of $\omega=10 \mathrm{rad} / \mathrm{s}$ in a constant field $\bar{B}=\hat{a}_{y} 30 \mathrm{mT}$. Determine the current induced in the loop in the direction shown, assuming the loop is in the $y z$ plane at $t=0$. The loop forms an angle $\alpha$ with respect to the $y z$ plane, where $\alpha=\omega t$.

$$
\text { (ans.: } V(t)=-0.015 \cos 10 t \mathrm{~A} \text { ) }
$$


(2) A $1-m^{2}$ wire loop completely encloses a time-varying magnetic field that is uniformly distributed over the interior surface of the loop, as shown in the figure below. A high-impedance voltmeter (one that draws negligible current) is placed in the three positions shown. Determine the voltmeter readings in each of the three positions.
(ans.: (a) -33.3 mV; (b) 66.7 mV ; (c) ?)
[9pts]

(b)

(a)

(c)
(3) A wheel of radius $R$ having a single spoke is rotating in a static magnetic field, magnitude $B$ at a speed of $N$ revolutions per second (rev/s) as shown in the figure below. Brushes are attached to the wheel axis and to the outer rim. Determine the voltage $V(t)$ induced between the brushes. The connection wires lie in a plane parallel to the magnetic field.

$$
\text { (ans.: } V(t)=-\pi N B R^{2} \mathrm{~V} \text { ) }
$$


(4) A PTFE-filled, parallel plate capacitor has plates of area $10^{-3} \mathrm{~m}^{2}$ that are separated by a distance of 0.04 mm . If the capacitor is connected to a $10 \mathrm{~V}, 15 \mathrm{MHz}$ sinusoidal voltage source, calculate the magnitudes of the displacement current, neglecting fringing fields, and the conduction current. Use $\epsilon_{r}=2.05$ for PTFE. (ans.: $I_{D}=427.7 \mathrm{~mA} ; I_{C}=$ ?)
[10 pts]
(5) Dry, sandy soil has $\sigma \approx 2.3 \times 10^{-9} \mathrm{~S} / \mathrm{m}$ and $\varepsilon_{r} \approx 3.45$ at 60 Hz , and $\sigma \approx 5.6 \times 10^{-3} \mathrm{~S} / \mathrm{m}$ and $\varepsilon_{r} \approx 2.5$ at 10 GHz . For distilled water, those values are $\sigma \approx 1.1 \times 10^{-8} \mathrm{~S} / \mathrm{m}$ and $\varepsilon_{r} \approx 81$ at 60 Hz , and $\sigma \approx 5.6$ $\mathrm{S} / \mathrm{m}$ and $\varepsilon_{r} \approx 50$ at 10 GHz . Copper, being an excellent conductor, has $\sigma \approx 5.8 \times 10^{7} \mathrm{~S} / \mathrm{m}$ and $\varepsilon_{r} \approx$ 1 from DC to well into the GHz reason. Calculate the ratio of the magnitude of the conduction current to the magnitude of the displacement current in these materials at both frequencies. (This ratio is the "loss tangent" $=\tan \delta$ of a material.) It will be sufficient to set up and solve the equation(s) needed to calculate the ratio for the first material and frequency, and then to provide a table of the all of the values.
(ans.: for dry, sandy soil, $\tan \delta=0.1997$ at $60 \mathrm{~Hz}, 0.00402$ at 10 GHz , etc.)

