

EE 382 Applied EM (Spring 2018) Examination #2

Name Key A

Instructions: Place answers in indicated spaces, use notation as given in class for coordinates & vectors, and **show & label all work** for credit. Express impedances & propagation constants in rectangular format and reflection coefficients & phasor currents/voltages in polar format (angle in degrees). Attach equation sheet and hand-in with exam.

- 1) A kimoyo bead technology load is connected to a lossless, 300Ω transmission line. Using an oscilloscope, the maximum and minimum peak-to-peak voltages on the transmission line are measured to be $5.4 V_{pp}$ and $1.5 V_{pp}$ respectively. Adjacent voltage minima occur at 20 cm intervals. There is 5.98 cm between the nearest voltage minima and load. Calculate the VSWR, wavelength, and operating frequency. Using a fully-labeled Smith chart, find the normalized impedance z_L and reflection coefficient of the load. Calculate the load impedance Z_L . Assume $u=c=2.998 \times 10^8$ m/s.

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{5.4}{1.5} = \underline{3.6}$$

$$\lambda = 20 \text{ cm} (2) = \underline{40 \text{ cm}} \leftarrow \text{adj. minima separated by } \frac{\lambda}{2}$$

$$f = \frac{u}{\lambda} = \frac{2.998 \times 10^8}{0.4} = \underline{749.5 \text{ MHz}}$$

→ Use VSWR scale on Smith Chart to set compass.
Draw circle of radius $VSWR=3.6$ ($|\Gamma|=0.565$) centered on Smith chart.

→ V_{min} occur @ $r_{min} = \frac{1}{VSWR} = 0.277$ point on circle

→ Move $\frac{5.98 \text{ cm}}{40 \text{ cm}} = 0.1495$ "WAVELENGTHS TOWARD LOAD" from r_{min} location to z_L / Γ_L point

$$\rightarrow \text{Read } \underline{z_L = 0.7 - j1.1 \Omega/\Omega}$$

$$\underline{\Gamma_L = 0.565 \angle -72.5^\circ}$$

$$\rightarrow Z_L = z_L Z_0 = (0.7 - j1.1) 300 = \underline{210 - j330 \Omega}$$

$$VSWR = \underline{3.6} \quad \lambda = \underline{40 \text{ cm}} \quad f = \underline{749.5 \text{ MHz}}$$

$$\Gamma_L = \underline{0.565 \angle -72.5^\circ} \quad z_L = \underline{0.7 - j1.1 \Omega/\Omega} \quad Z_L = \underline{210 - j330 \Omega}$$

1) continued

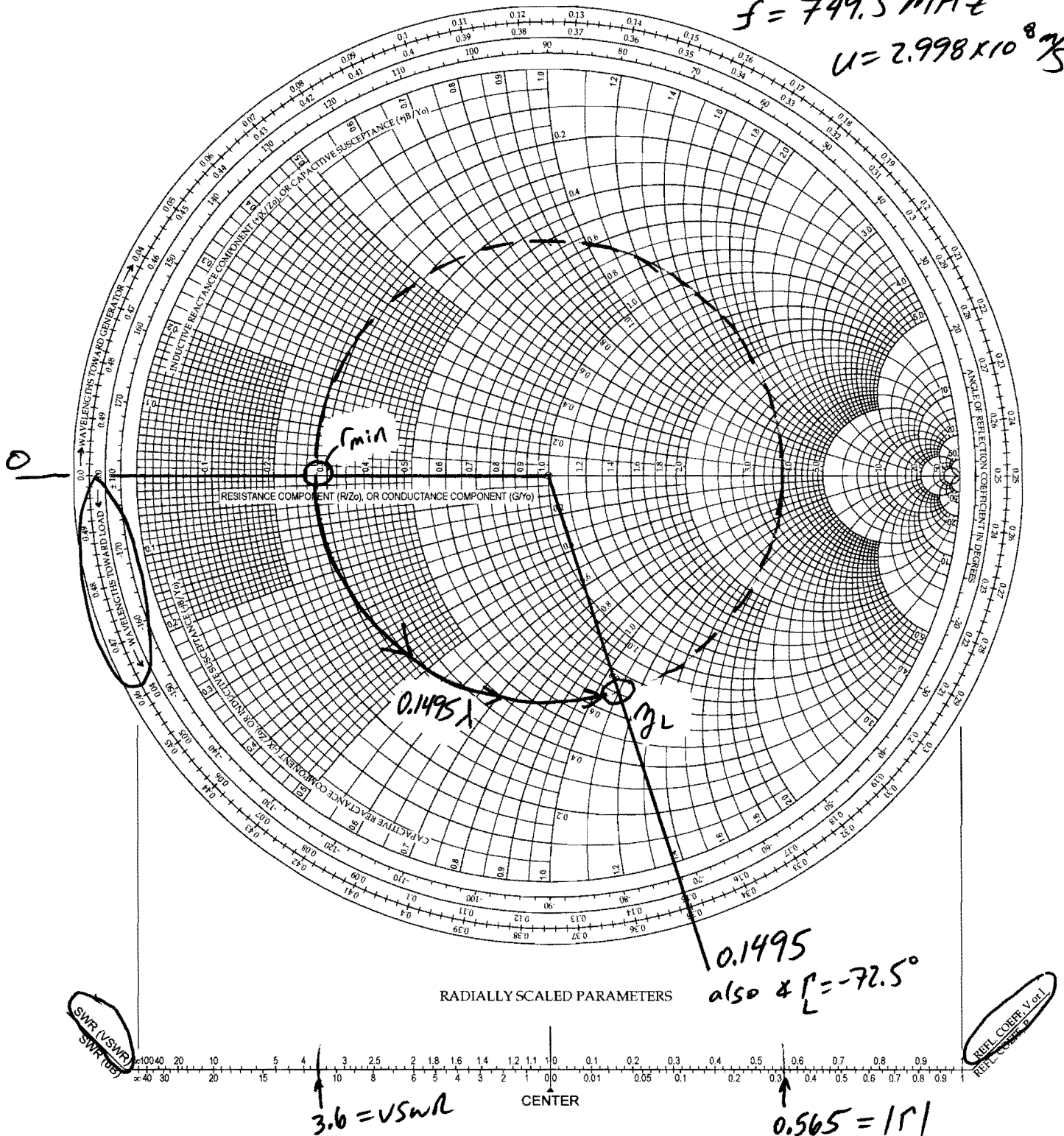
Simple Smith Chart

$Z_0 = 300 \Omega$

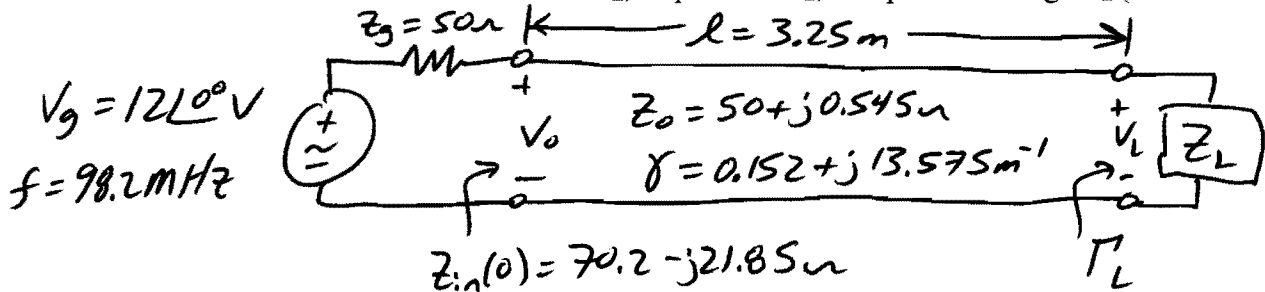
$\lambda = 40 \text{ cm}$

$f = 749.5 \text{ MHz}$

$v = 2.998 \times 10^8 \text{ m/s}$



2) A 3.25 m long vibranium transmission line has $Z_0 = 50 + j0.545 \Omega$ and $\gamma = 0.152 + j13.575 \text{ m}^{-1}$. It is driven by a generator, operating at 98.2 MHz, with $V_g = 12\angle 0^\circ \text{ V}$ and $Z_g = 50 \Omega$. Using a vector network analyzer, the input impedance is measured to be $Z_{in}(0) = 70.2 - j21.85 \Omega$. Find the input reflection coefficient $\Gamma_{in}(0)$, phasor input voltage V_0 , and forward traveling voltage wave V_0^+ . Determine the load reflection coefficient Γ_L , impedance Z_L , and phasor voltage V_L (extra credit).



$$\Gamma_{in}(0) = \frac{Z_{in}(0) - Z_0}{Z_{in}(0) + Z_0} = \frac{(70.2 - j21.85) - (50 + j0.545)}{(70.2 - j21.85) + (50 + j0.545)} = 0.24706 \angle -37.899^\circ$$

$$V_0 = V_g \frac{Z_{in}(0)}{Z_g + Z_{in}(0)} = (12\angle 0^\circ) \frac{70.2 - j21.85}{75 + (70.2 - j21.85)} = 7.2216 \angle -6.986^\circ \text{ V}$$

Note: $V_0 = V_0^+ e^0 (1 + \Gamma_{in}(0))$

$$\hookrightarrow V_0^+ = \frac{V_0}{1 + \Gamma_{in}(0)} = \frac{7.2216 \angle -6.986^\circ}{1 + 0.2471 \angle -37.899^\circ} = 5.9953 \angle 0.25175^\circ \text{ V}$$

$$\Gamma_L = \Gamma_{in}(0) e^{2\gamma l} = (0.2471 \angle -37.899^\circ) e^{2(0.152 + j13.575)3.25}$$

$$\Gamma_L = 0.66356 \angle -22.26265^\circ$$

$$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L} = (50 + j0.545) \frac{1 + 0.6636 \angle -22.26^\circ}{1 - 0.6636 \angle -22.26^\circ}$$

$$Z_L = 133.22 - j117.08 \Omega$$

Extra Credit

$$V_L = V_0^+ e^{-\gamma l} (1 + \Gamma_L) = (5.9953 \angle 0.25175^\circ) e^{-(0.152 + j13.575)3.25} (1 + 0.6636 \angle -22.26^\circ)$$

$$V_L = 5.976 \angle -16.42^\circ \text{ V}$$

$$\Gamma_{in}(0) = 0.2471 \angle -37.899^\circ \quad V_0 = 7.222 \angle -6.986^\circ \text{ V} \quad V_0^+ = 5.995 \angle 0.252^\circ \text{ V}$$

$$\Gamma_L = 0.6636 \angle -22.26^\circ \quad Z_L = 133.22 - j117.08 \Omega \quad \boxed{V_L = 5.976 \angle -16.42^\circ \text{ V} \text{ (Extra)}}$$

- 3) A load $Z_L = 90 + j210 \Omega$ is connected to a 300Ω , $8\angle 0^\circ$ V, 800 MHz generator by a 90 cm long, lossless transmission line ($u = 2.6 \times 10^8$ m/s, $Z_0 = 300 \Omega$) in a Royal Talon fighter sensor module. Using a Smith chart, determine the input impedance $Z_{in,NM}$ and $VSWR_{NM}$ with no matching. Next, using a Smith chart, match the load to the generator using the shortest possible **open circuit** stub, made of the same transmission line, connected in parallel as close to the load as possible. What are the normalized admittances of the closest match point y_{M1} and the corresponding stub y_{stub} ? How long is the stub d_{stub} (in λ & cm) and how far is the stub from the load l_{stub} (in λ & cm)? Sketch the resulting circuit with all relevant lengths (cm) and values labeled. **Clearly label all points and relevant dimensions/parameters on Smith chart.**

No Match

→ Normalize load impedance $y_L = \frac{Z_L}{Z_0} = \frac{90 + j210}{300} = 0.3 + j0.7 \frac{\Omega}{\Omega}$

→ Plot y_L on Smith Chart

→ Draw circle, centered on Smith chart, through y_L

→ Move $\frac{l}{\lambda} = \frac{0.9}{\left(\frac{2.6 \times 10^8}{800 \times 10^6}\right)} = \frac{0.9}{0.325} = 2.76923 \rightarrow 0.26923$

"WAVELENGTHS TOWARD GENERATOR" from y_L to

$$\underline{y_{in,NM} = 0.4 - j0.97 \frac{\Omega}{\Omega}}$$

→ $Z_{in,NM} = y_{in,NM} Z_0 = (0.4 - j0.97)300 = \underline{120 - j291 \Omega}$

→ Use compass + VSWR scale to get $VSWR_{NM} \approx 5.05$
(Exact 5.07)

Matching

→ Move 180° (or $\lambda/4$) around circle from y_L to

$$y_L = 0.517 - j1.207 \frac{\Omega}{\Omega} \text{ point}$$

→ From y_L , need to move $(0.1482 + 0.183)\lambda = \underline{0.3312 \lambda = l_{STUB}}$

"WAVELENGTHS TOWARD GENERATOR" to arrive

$$\text{@ first match point } \underline{y_{M1} = 1 + j1.8 \frac{\Omega}{\Omega}}$$

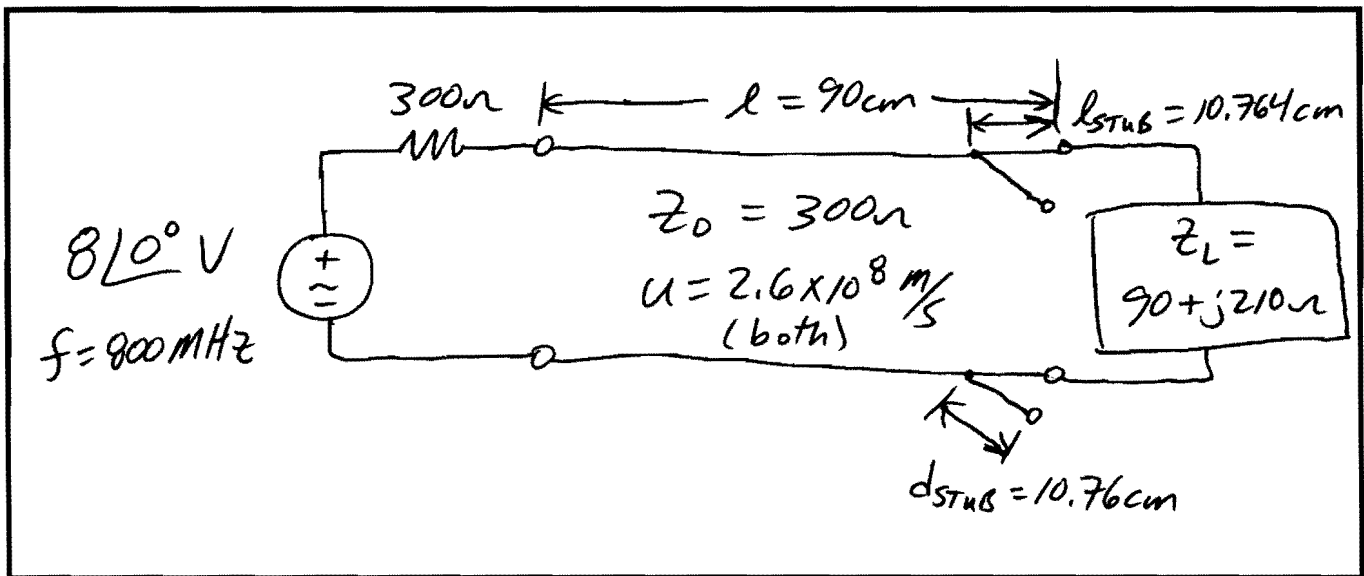
3) continued

→ To match $y_{in} + y_{stub} = 1 = (1 + j1.8) + y_{stub}$

↳ $y_{stub} = -j1.8 \text{ S}$

→ Starting @ open ckt point ($y_{oc} = \frac{1}{\infty} = 0$) on far left of Smith Chart ($\Gamma_{oc} = 1 \angle +180^\circ = +1$), move along perimeter ($|\Gamma|=1$) of Smith Chart toward generator to arrive @ $y_{stub} = -j1.8 \text{ S}$.
 Reading scale $d_{stub} = 0.331 \lambda = 0.331(32.5)$
 $= 10.7575 \text{ cm}$

Fully-labeled sketch of **matched** transmission line circuit



VSWR_{NM} = 5.07

Z_{in,NM} = 120 - j291 Ω

y_{MI} = 1 + j1.8 S/m

y_{stub} = -j1.8 S

d_{stub} = 0.331 λ = 10.7575 cm

l_{stub} = 0.3312 λ = 10.764 cm

3) continued

Simple
Smith Chart

$Z_0 = 300 \Omega$
 $u = 2.6 \times 10^8 \text{ m/s}$
 $f = 800 \text{ MHz}$
 $\lambda = 32.5 \text{ cm}$

