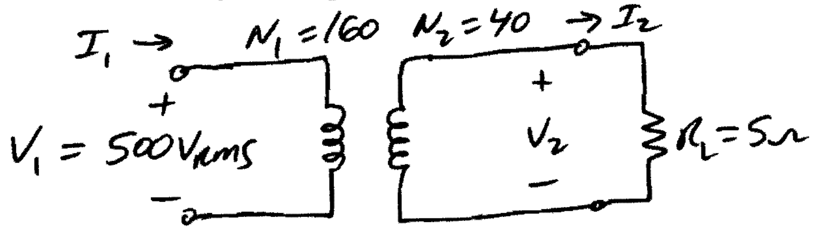
EE 382 Applied Electromagnetics Examination #1 (Spring 2018)

Name Key A

Instructions: Place answers in indicated spaces. Use notation as given in class for coordinates and vectors. Show all work for full credit. Turn in equation sheet with exam.

1) An ideal transformer has 160 turns on the primary winding and 40 turns on the secondary winding. If 500 V_{rms} is applied to the primary winding and an equivalent load resistance of 5 Ω is connected to the secondary winding, sketch circuit and find:



a) the RMS voltage supplied to the load

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \Rightarrow V_2 = \frac{N_2}{N_1} V_1 = \frac{40}{160}(500) = 125\ V_{rms}$$

RMS voltage to load = 125 V_{rms}

b) the RMS current supplied to the load

$$I_2 = V_2 / R_L = \frac{125}{5} = 25\ A_{rms}$$

RMS current to load = 25 A_{rms}

c) the RMS current through the primary winding

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \Rightarrow I_1 = I_2 \left(N_2 / N_1 \right) = 25 \left(\frac{40}{160} \right) = 6.25\ A_{rms}$$

RMS current through primary = 6.25 A_{rms}

d) the equivalent resistance seen on the primary side of the transformer

$$R_{1,eff} = \left(\frac{N_1}{N_2}\right)^2 R_L = \left(\frac{160}{40}\right)^2 5 = 80\Omega$$

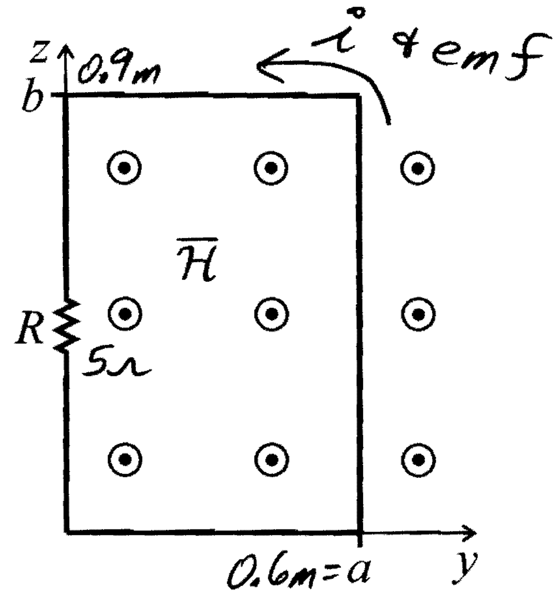
$$= V_1 / I_1 = \frac{500}{6.25} = 80\Omega$$

equivalent resistance seen on primary = 80Ω

e) Is this a step-up, step-down, or isolation/unity transformer? (Circle correct answer)

2) A wire loop (see below) in air has the magnetic field $\bar{\mathcal{H}} = \hat{a}_x 800(1+0.5z)e^{-600t}$ (A/m) $t > 0$ applied. Find and indicate on the drawing the direction of **positive** current flow (Hint: Lenz's Law). Choosing an integration contour in the same direction as the current, calculate and write-out expressions for the magnetic flux through the loop, and the current and *emf* around the loop. Given: $a = 60$ cm, $b = 90$ cm, $R = 5\ \Omega$, $\mu_0 = 4\pi\times10^{-7}$ H/m.

$|\bar{\mathcal{H}}|$ decreases wrt to time per e^{-600t} term. Therefore, Lenz's Law states the induced emf will cause i to flow CCW to produce an induced $\mathcal{H}$ that opposes the decrease.



As $\hat{a}_y \times \hat{a}_z = +\hat{a}_x$ out of page

$$\Psi_m = \iint_S \bar{B}\cdot d\bar{s} = \iint_S \mu_0 \bar{\mathcal{H}}\cdot d\bar{s}_x = \iint_S \mu_0\, \hat{a}_x 800(1+0.5z)e^{-600t}\cdot d\bar{s}_x$$

$$= 800\mu_0 e^{-600t}\int_{z=0}^{0.9}(1+0.5z)dz\int_{y=0}^{0.6}dy = 800\mu_0 e^{-600t}\left[z+\frac{z^2}{4}\right]_0^{0.9}(y)\Big|_0^{0.6}$$

$$= 800\mu_0 e^{-600t}\left[\left(0.9+\frac{0.9^2}{4}\right)-(0+0)\right](0.6-0) = \underline{665.012\times10^{-6}\, e^{-600t}\ (\text{Wb})}$$

Faraday's Law

$$emf = -\frac{\partial \Psi_m}{\partial t} = -665.012\times10^{-6}(-600)e^{-600t} = \underline{0.3990074\, e^{-600t}\ V}$$

Ohm's Law

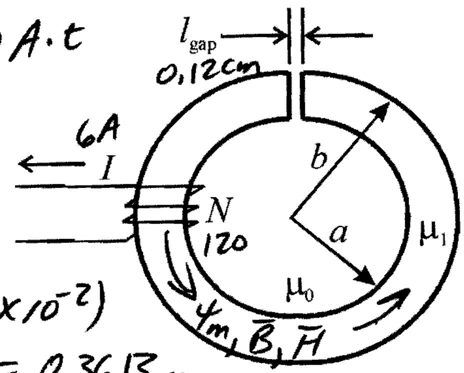
$$i = \frac{emf}{R} = \frac{0.3990074\, e^{-600t}}{5} = \underline{0.0798015\, e^{-600t}\ A}$$

magnetic flux = $665.012\, e^{-600t}$ μWb $t > 0$

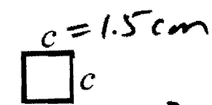
emf = $399.007\, e^{-600t}$ mV $t > 0$ $i(t)$ = $79.8015\, e^{-600t}$ mA $t > 0$

3) For the split magnetic ring shown, you are given $a = 5$ cm, $b = 6.5$ cm, $c = 1.5$ cm, $N = 120$, $I = 6$ A, and $l_{gap} = 0.12$ cm, and $\mu_1 = 500\mu_0$. First, sketch a **fully labeled** equivalent magnetic circuit. Then, find the *magnitude* of the magnetic flux, flux density, and field in core material 1. Indicate their direction on the drawing. Also, find the *mmf* across the top air gap. Assume the magnetic field is evenly distributed in the core, the coil is tightly wound, no flux leakage, and no fringing in gaps.

$\mathcal{F}_{coil} = NI = 120(6) = 720\ A\cdot t$



Core
Cross-section



Ave Radius $= \dfrac{a+b}{2} = \dfrac{5+6.5}{2}$

$= 5.75$ cm

$A_{core} = c^2 = 2.25 \times 10^{-4}\ m^2$

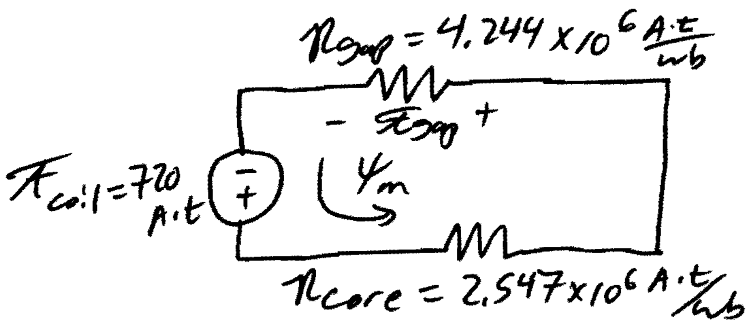
ave circumference $= 2\pi(5.75 \times 10^{-2})$

$= 0.115\pi = 0.3613 m$

$l_{core} = $ Ave. Circum. $- l_{gap} = 0.115\pi - 0.12 \times 10^{-2} = 0.36008315 5m$

$$\mathcal{R}_{core} = \frac{l_{core}}{\mu_1 A_{core}} = \frac{0.36008}{500(4\pi \times 10^{-7})(1.5 \times 10^{-2})^2} = 2{,}547{,}067.292\ \frac{A\cdot t}{wb}$$

$$\mathcal{R}_{gap} = \frac{l_{gap}}{\mu_0 A_{core}} = \frac{0.12 \times 10^{-2}}{4\pi \times 10^{-7}(1.5 \times 10^{-2})^2} = 4{,}244{,}131.816\ \frac{A\cdot t}{wb}$$



$$\psi_m = \frac{\mathcal{F}_{coil}}{\mathcal{R}_{gap} + \mathcal{R}_{core}} = \frac{720}{6791199.11}$$

$$= \underline{1.0601957 \times 10^{-4}\ Wb}$$

$$B_{core} = \frac{\psi_m}{A_{core}} = \frac{1.0602 \times 10^{-4}}{(1.5 \times 10^{-2})^2} = \underline{0.4712\ Wb/m^2}$$

$$H_{core} = \frac{B_{core}}{\mu_{core}} = \frac{0.4712}{500(4\pi \times 10^{-7})} = \underline{749.935\ A/m}$$

$$mmf_{gap} = \mathcal{F}_{gap} = \psi_m \mathcal{R}_{gap} = 1.06 \times 10^{-4}(4.244 \times 10^6) = \underline{449.961\ A\cdot t}$$

$|$mag. flux$|_1$ = $106.02\ \mu Wb$ $|$mag. flux density$|_1$ = $0.4712\ Wb/m^2$

$|$magnetic field$|_1$ = $749.935\ A/m$ mmf_{gap} = $449.961\ A\cdot t$

4) In a barium titanate composite (μ_0, $300\varepsilon_0$, and $\sigma \approx 0$), we apply an electric field of $\vec{\mathcal{E}} = \hat{a}_y 8\cos(5{\cdot}10^8 t - kx)$ (kV/m) where $k = \omega/u = 2\pi/\lambda$ and $u = c/\sqrt{\varepsilon_r}$. Determine the wave number k and vector phasor electric field $\bar{E}$ with all constants defined. Then, use $\bar{E}$ to find the vector time-domain magnetic field $\bar{\mathcal{H}}$ with all constants defined. For extra credit, find the displacement current density $\bar{\mathcal{J}}_d$. [Hints: Faraday & no free charges are present.]

$$\omega = 5\times10^8 \tfrac{rad}{s}, \quad u = \frac{c}{\sqrt{\epsilon_r}} = \frac{2.99792458\times10^8}{\sqrt{300}} = 1.73085\times10^7 \ m/s$$

$$K = \omega/u = \frac{5\times10^8}{1.731\times10^7} = \underline{28.8875 \ rad/m}$$

$$\underline{\bar{E} = \hat{a}_y 8 e^{-j28.8875x} \ kV/m}$$

Phasor Faraday's Law $\bar{\nabla}\times\bar{E} = -j\omega\bar{B} = -j\omega\mu_0\bar{H}$

$$\bar{\nabla}\times\bar{E} = \left[\frac{\partial \cancel{E_z}^{0}}{\partial y} - \frac{\partial \cancel{E_y}}{\partial z}\right]\hat{a}_x + \left[\frac{\partial \cancel{E_x}^{0}}{\partial z} - \frac{\partial \cancel{E_z}^{0}}{\partial x}\right]\hat{a}_y + \left[\frac{\partial E_y}{\partial x} - \frac{\partial \cancel{E_x}^{0}}{\partial y}\right]\hat{a}_z = -j\omega\mu_0\bar{H}$$

$\xrightarrow{}0$ $\qquad\qquad$ 5×10^8 $\quad$ $4\pi\times10^{-7}$

$$\hat{a}_z \frac{\partial}{\partial x}\left(8000 e^{-j28.8875x}\right) = -j\omega\mu_0\bar{H}$$

$$\hat{a}_z 8000 e^{-j28.8875x}(-j28.8875) = -j\,5\times10^8(4\pi\times10^{-7})\bar{H}$$

$$\hookrightarrow \bar{H} = \hat{a}_z \frac{-j28.8875(8000)}{-j\,5\times10^8(4\pi\times10^{-7})} e^{-j28.8875x} = \hat{a}_z 367.8071 e^{-j28.8875x} \ A/m$$

$$\underline{\bar{\mathcal{H}} = \mathcal{R}e\{\bar{H}e^{j\omega t}\} = \hat{a}_z 367.807\cos(5\times10^8 t - 28.8875x) \ A/m}$$

$$\bar{\mathcal{J}}_d = \frac{\partial \bar{\mathcal{D}}}{\partial t} = 300\epsilon_0 \frac{\partial \bar{\mathcal{E}}}{\partial t} = 300\epsilon_0 \hat{a}_y 8000(-\sin(5\times10^8 t - 28.8875x))\,5\times10^8$$

$$= \underline{-\hat{a}_y 10625.025\sin(5\times10^8 t - 28.8875x) \ A/m^2}$$

$\bar{E}$ = $\hat{a}_y 8 e^{-j28.8875x}$ kV/m $\qquad$ k = 28.8875 rad/m

$\bar{\mathcal{H}}$ = $\hat{a}_z 367.807\cos(5\times10^8 t - 28.8875x)$ A/m

(extra credit) $\bar{\mathcal{J}}_d$ = $-\hat{a}_y 10.625\sin(5\times10^8 t - 28.8875x)$ kA/m²