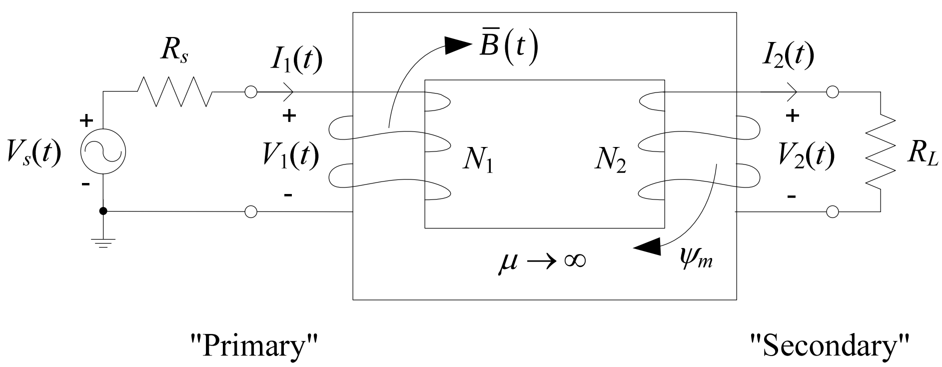
Lecture 9: Ideal Transformer.

In general, a transformer is an n-port AC device (sometimes a two-port device) that converts time varying voltages and currents from one amplitude at an input port to other values at the output ports. This also has the effect of transforming impedance levels. This device only performs this transformation for time varying signals.

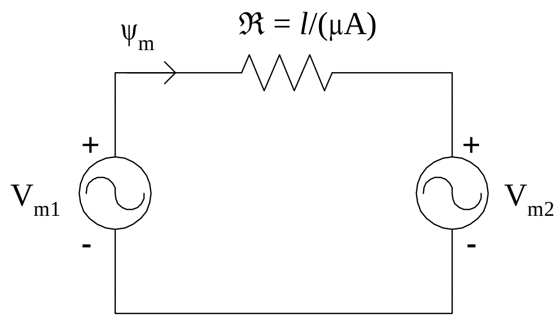
Here, we will consider the two-port transformer circuit shown below:



Notice that there is no continuous path for conduction current from the voltage source V_s to the load resistor R_L. These two ports instead are electrically "coupled" to each other indirectly by the principal of induction from Faraday's law.

That is, a time varying current from the source creates a time varying magnetic flux in the primary coil that travels through the core to the secondary coil, which then creates a time-varying voltage in the secondary terminals, again by Faraday's law.

We will analyze this physical transformer as a ***time varying*** magnetic circuit (ignoring flux leakage):



From this magnetic circuit we find

$$V_{m1}(t) - V_{m2}(t) = \Re \psi_m(t) \tag{1}$$

where $V_{m1}(t) = N_1 I_1(t)$, $V_{m2}(t) = N_2 I_2(t)$ and $\Re = \dfrac{l}{\mu A}$.

Substituting these into (1) gives

$$N_1 I_1(t) - N_2 I_2(t) = \frac{l}{\mu A} \psi_m(t) \tag{2}$$

In an **ideal transformer**:

- the core permeability μ is linear wrt the magnetic flux,
- $\mu \to \infty$, and
- the windings are perfect conductors.

From the second of these ideal transformer assumptions, the RHS of (2) vanishes leaving

$$N_1 I_1(t) - N_2 I_2(t) = 0$$

or

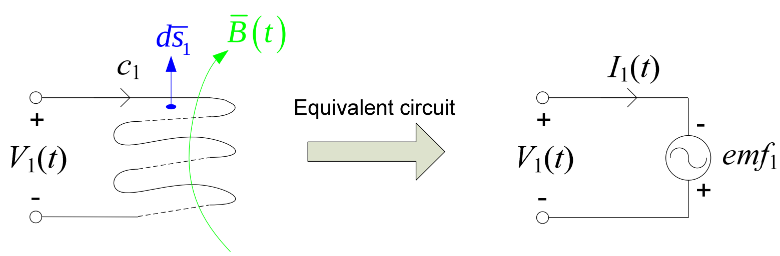
$$\boxed{\frac{I_1(t)}{I_2(t)} = \frac{N_2}{N_1}} \tag{3}$$

Furthermore, by Faraday's law we know that for a coil with N identical turns of wire

$$emf = -N\frac{d\psi_m}{dt} \tag{4}$$

where ψ_m is now the magnetic flux through just one (identical) turn of wire. (Sometimes this relationship is written in terms of so called *flux linkage* as $\lambda = N\psi_m$.)

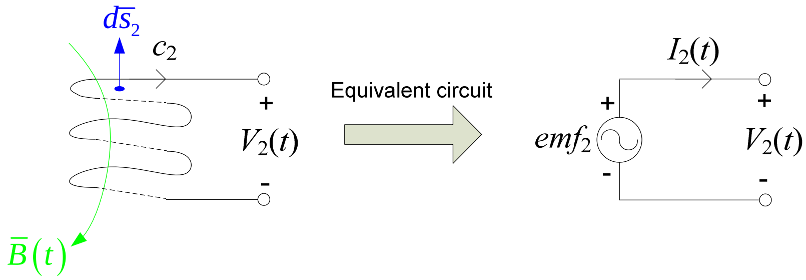
For ease of reference, the primary coil circuit from the transformer shown on page 1 is sketched below. An equivalent electrical circuit for this primary coil is also shown.



Given the direction for $\bar{B}$ and the assumed direction for c_1 (which gives rise to the direction of $d\bar{s}_1$), then from the equivalent circuit and (4) we find

$$V_1(t) = -emf_1 \underbrace{=}_{\bar{B}\cdot d\bar{s}_1 > 0} N_1\frac{d\psi_m}{dt} \tag{5}$$

Similarly, for the secondary coil circuit the equivalent circuit is



Notice here that the direction of $d\bar{s}_2$ (dictated by the given direction for I_2 and hence c_2) is <u>opposite</u> that of $\bar{B}$. Consequently, from the equivalent circuit and using (4) again we determine that

$$V_2(t) = +emf_2 \underbrace{=}_{\bar{B}\cdot d\bar{s}_2<0} N_2 \frac{d\psi_m}{dt} \tag{6}$$

Now, provided $\dfrac{d\psi_m}{dt} \neq 0$ (because a transformer does not "transform" at DC), the ratio of (5) and (6) gives

$$\boxed{\frac{V_1(t)}{V_2(t)} = \frac{N_1}{N_2}} \tag{7}$$

Equations (3) and (7) are the basic equations of an **ideal transformer**.

Discussion

1. From (7), the voltage at the so-called "secondary" of the transformer is

$$V_2(t) = \frac{N_2}{N_1} V_1(t) \tag{8}$$

Note that if $N_2 > N_1$, the secondary voltage is larger than the primary voltage! Very interesting.

- If $N_2 > N_1$, called a step-up transformer,
- If $N_2 < N_1$, called a step-down transformer.

2. From (3), the secondary current is

$$I_2(t) = \frac{N_1}{N_2} I_1(t) \tag{9}$$

We can surmise from (9) that for a step-up transformer, $I_2(t) < I_1(t)$. Therefore, while the voltage increases by N_2 / N_1, the current has decreased by N_1 / N_2.

Because of this property, the power input to the primary equals the power output from the secondary:

$$P_1(t) = V_1(t) I_1(t) \tag{10}$$

$$P_2(t) = V_2(t) I_2(t) = \underbrace{\frac{N_2}{N_1} V_1(t)}_{(8)} \cdot \underbrace{\frac{N_1}{N_2} I_1(t)}_{(9)} = V_1(t) I_1(t) \tag{11}$$

Therefore, the input power $P_1(t)$ equals the output power $P_2(t)$.

3. With a resistance R_L connected to the secondary, then

$$\frac{V_2(t)}{I_2(t)} = R_L$$

Substituting for V_2 and I_2 from (8) and (9)

$$\frac{\dfrac{N_2}{N_1}\cdot V_1(t)}{\dfrac{N_1}{N_2}\cdot I_1(t)} = R_L$$

or

$$\frac{V_1(t)}{I_1(t)} = \left(\frac{N_1}{N_2}\right)^2 R_L$$

In other words, the **effective input resistance** $R_{1,eff}$ at the primary terminals (the ratio V_1/I_1) is

$$\boxed{R_{1,eff} = \left(\frac{N_1}{N_2}\right)^2 R_L} \tag{12}$$

The transformer "transforms" the load resistance from the secondary to the primary. (Remember that this is only true for time varying signals.)

For sinusoidal steady state and load impedance Z_L, equation (12) becomes

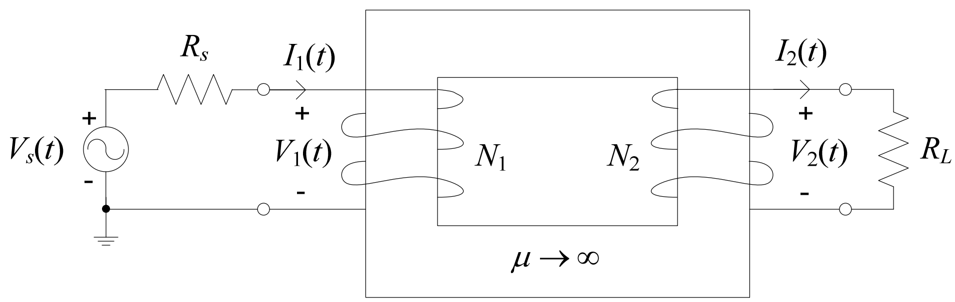
$$\boxed{Z_{1,eff} = \left(\frac{N_1}{N_2}\right)^2 Z_L} \tag{13}$$

4. For maximum power transfer, we design a circuit so that the load is matched to the output resistance. We can use transformers as "matching networks."

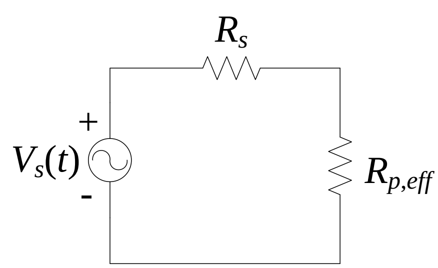
5. Notice that the primary has the source connection so that the ground occurs at the "-" V_s terminal. However, the secondary is not grounded. This secondary is said to be "balanced." (An exception to this is the autotransformer.)

6. Remember that only time varying signals are "transformed" by a transformer.

Example N10.1: Design the transformer shown below so that maximum power is delivered to the load R_L for fixed R_s and R_L.



This transformer "transforms" the load resistance to the primary according to (12). An equivalent circuit at the primary terminal can be constructed using this effective primary resistance:



From (12)
$$R_{p,eff} = \left(\frac{N_1}{N_2}\right)^2 R_L$$

(As an aside, note that $R_{p,eff} \to \infty$ as $R_L \to \infty$, which is an open circuit. In practical transformers, it's not uncommon for $I_1(t) \approx$ some small fraction of rated I for an open load.)

For maximum power transfer $R_{p,eff} = R_s^*$. Consequently,
$$\left(\frac{N_1}{N_2}\right)^2 R_L = R_s \quad \text{or} \quad \frac{N_1}{N_2} = \sqrt{\frac{R_s}{R_L}}$$
The "turns ratio" N_1/N_2 is adjusted to this value for maximum power transfer from the source to R_L, even when $R_L \neq R_s$.