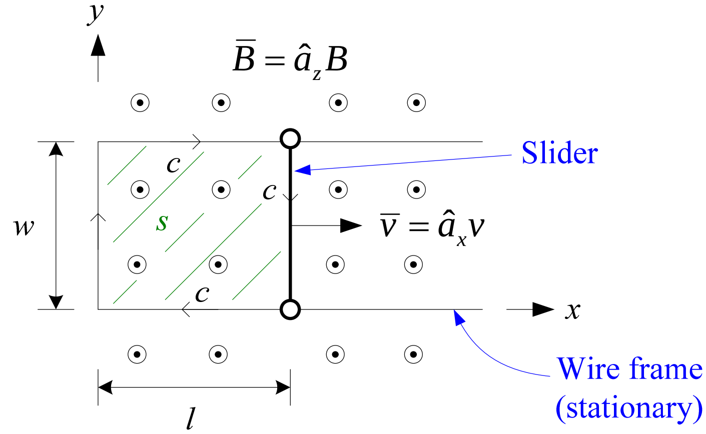
Lecture 3: Faraday's Law Examples.

We will now solve a number of examples that involve Faraday's law.

Example N3.1: Determine the emf around the contour *c* that includes the moving slider in the figure below.



Apply Faraday's law to the contour *c* in the direction shown:

$$emf = -\frac{d\psi_m}{dt} = -\frac{d}{dt}\int_{s(c)} \overline{B}\cdot d\overline{s}$$

ψ_m changes with time because the surface *s* increases with *t* as the slider moves to the right. The magnetic flux density $\overline{B}$, however, is **not** changing with time.

Therefore,

$$emf = -\frac{d}{dt}\int_s (\hat{a}_z B)\cdot(-\hat{a}_z)\,dxdy$$
$$= B\frac{d}{dt}\int_0^w\int_0^l dxdy = wB\underbrace{\frac{dl}{dt}}_{=v} = wBv \ \ \text{[V]} \qquad (1)$$

Note that we have ignored $\bar{B}_{\text{ind}}$ created by the current induced in the wire. This would be a reasonable assumption for a high resistance wire, for example.

Another way to approach this problem is with the Lorentz force equation

$$\bar{F} = q\left(\bar{E} + \bar{v}\times\bar{B}\right)$$

In this problem, $\bar{F}_e = 0$ while

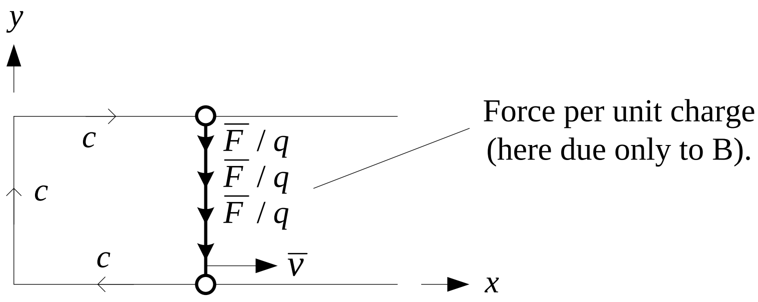
$$\frac{\bar{F}_m}{q} = \begin{cases} -\hat{a}_y vB & \text{on slider} \\ 0 & \text{elsewhere} \end{cases}$$

Then,

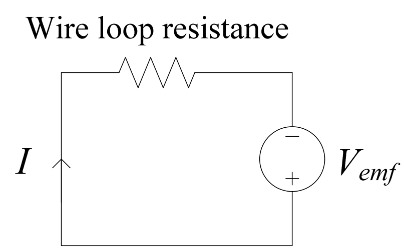
$$emf \equiv \oint_c \bar{E}\cdot d\bar{l} = \oint_c \frac{\bar{F}}{q}\cdot d\bar{l} = \oint_c \frac{\bar{F}_m}{q}\cdot d\bar{l}$$
$$= \int_{\text{slider}} \left(-\hat{a}_y vB\right)\cdot\left(-\hat{a}_y dy\right) = wvB \ \ \text{[V]} \qquad (2)$$

We see from (1) and (2) that both approaches give the same result, as would be expected.

Physically, this emf will cause charges to move in the wire since $\bar{F}/q = \bar{v} \times \bar{B}$ (like an electric field!):



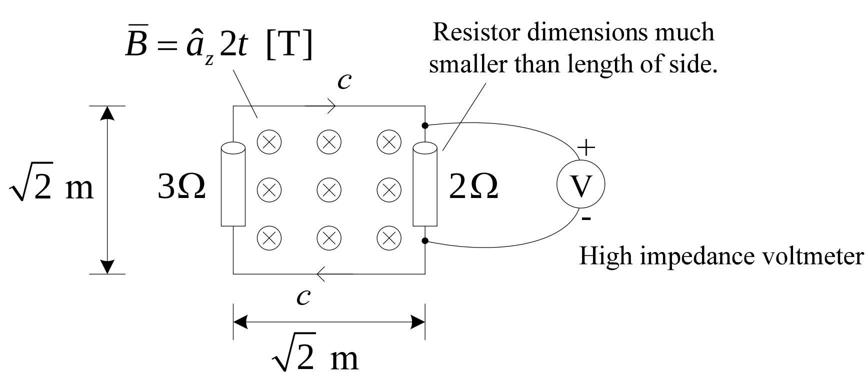
The **equivalent circuit model** for this is:



Note in the construction of this equivalent circuit that:

1. positive current is in the direction of positive emf,
2. current enters the negative terminal of the source.

Example N3.2: Determine the voltage measured by the high-impedance voltmeter in the circuit below.



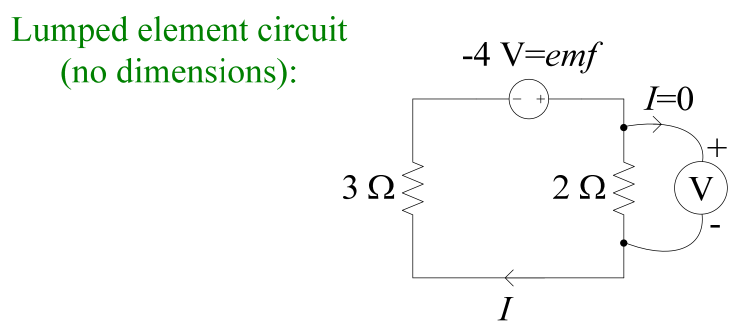
Interestingly, this circuit has no traditional source in it such as a voltage source or current source. So how can the voltmeter read anything but zero? The answer is by Faraday's law: an induced emf will cause a current to flow in the circuit!

For the direction of c shown,

$$emf = -\frac{d\psi_m}{dt} = -\frac{d}{dt}\int_{s(c)} \bar{B}\cdot d\bar{s} = -\frac{d}{dt}A\cdot B_z$$

$$= -\frac{d}{dt}\left(\sqrt{2}\right)^2 \cdot \underbrace{(2t)}_{d\bar{s}\text{ points into page}} = -4 \text{ [V]}$$

This *emf* serves as a voltage source in the **equivalent lumped-element** circuit:



It is very important to note the polarity of the equivalent emf voltage source in this circuit. The contour c was initially chosen clockwise. Consequently, the current I must enter the '-' terminal of this voltage source, as shown.

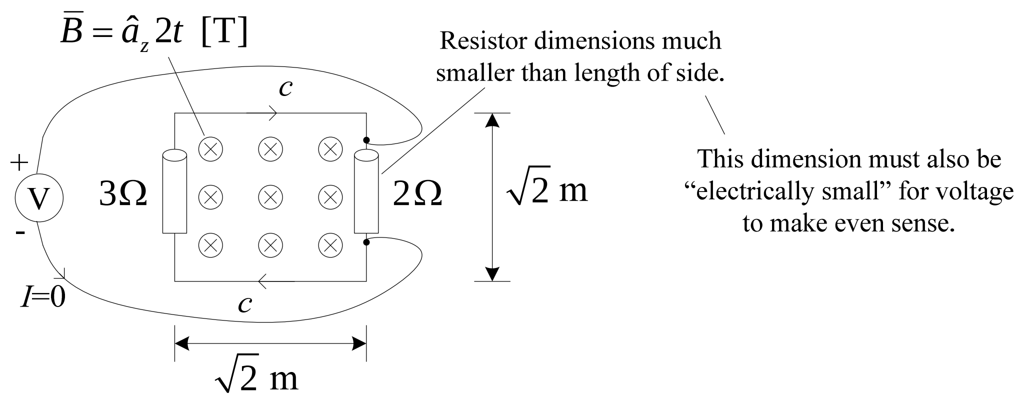
Therefore,

$$I=\frac{emf}{3+2}=-\frac{4}{5}\ \text{[A]}\quad\Rightarrow\quad V=-2\cdot\frac{4}{5}=-\frac{8}{5}\ \text{[V]}$$

Summary of steps for the solution:

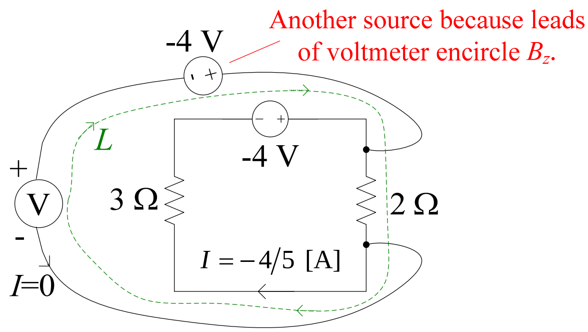
1. Begin with the actual physical circuit, the one with dimensions.
2. Pick a direction for the contour c (used in the emf equation).
3. Compute $emf\ (=-d\psi_m/dt)$.
4. Construct the lumped element circuit (no physical dimensions) inserting the appropriate equivalent emf source(s).
5. Solve the lumped element circuit for the desired voltage(s) and current(s) using traditional electrical circuit methods.

Example N3.3: Determine the voltage measured in the circuit of the previous example, but with the high-impedance voltmeter and leads oriented as shown below.



Our expectation might be that there won't be any change to the measured voltage. How could it since we've only changed how the leads of the high-impedance voltmeter are laid out. By Faraday's law and induced emf, however, we will see that there **is** a change to the measured voltage.

The equivalent lumped element circuit is:



Using KVL around loop L gives:

$$V + (-4) = I \cdot 2$$

or

$$V = -\frac{4}{5} \cdot 2 + 4 = \frac{12}{5} \text{ [V]}$$

Something very strange has just happened. The measured voltages in these last two example problems have very different values ($-8/5$ V and $12/5$ V)!

Voltage May Not Be Unique

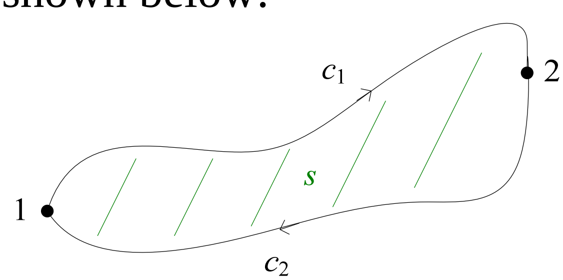
This is an example illustrating that **"voltage" may not be a unique quantity** for time-varying electromagnetic fields. Measured voltages may depend on how the leads of the voltmeter (or oscilloscope) are laid out and the time-rate-of-change of the magnetic flux through this measuring loop.

To be more specific, why was the measured voltage different in the previous two examples? Because electric scalar potential Φ_e is **not unique** for time-varying fields.

Consider Faraday's law

$$\oint_{c(s)} \overline{E} \cdot d\overline{l} = -\frac{d}{dt} \int_{s(c)} \overline{B} \cdot d\overline{s} \tag{1}$$

and the contours shown below:



Along c, (1) is

$$\int_{c_1} \bar{E} \cdot d\bar{l} + \int_{c_2} \bar{E} \cdot d\bar{l} = -\frac{d}{dt} \int_{s(c)} \bar{B} \cdot d\bar{s} \tag{2}$$

$$\neq 0$$

This is generally not zero unless:

- no time variation, or
- no magnetic flux linkage through *s*.

Therefore, we conclude that $\bar{E}(t)$ is **not conservative**. Since it is not conservative, we cannot define a scalar potential as $\bar{E} = -\nabla\Phi_e$ as was done in Ch. 4 of the text (at least not uniquely).

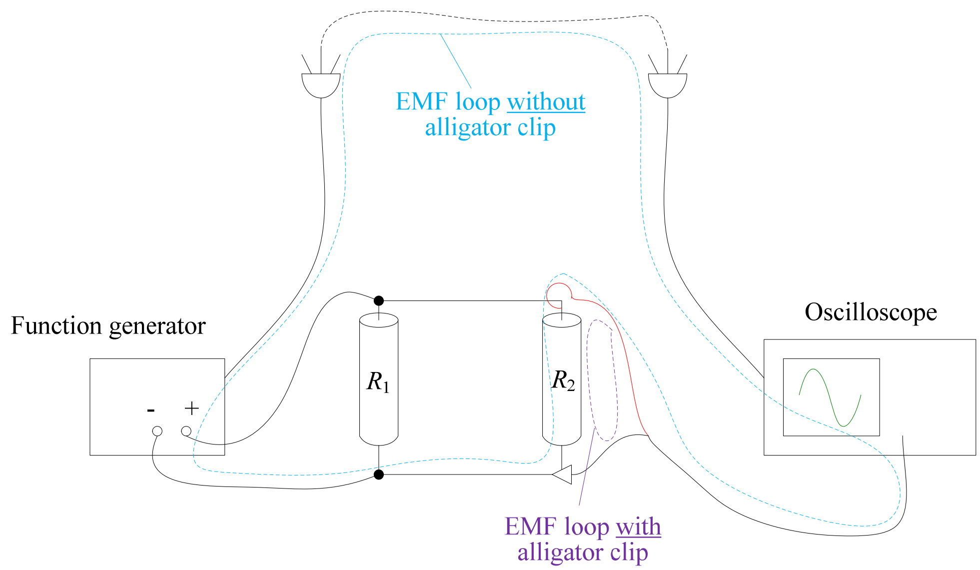
But, if we **did** define

$$\Delta\Phi_e = -\int_a^b \bar{E} \cdot d\bar{l}$$

we can deduce from (2) that $\Delta\Phi_e$ depends on the contour taken, and consequently $\Delta\Phi_e$ is **not unique**! You will generally compute (or measure) a different voltage depending on the path used for the integration (or how the measurement leads are laid out)!

At "high" frequencies, this non-uniqueness of scalar electric potential can adversely affect oscilloscope measurements.

A typical oscilloscope is **single-ended input** meaning that the black alligator clip on the scope probe is earth ground.



Measurements taken without connecting the scope probe alligator clip to ground will be correct at "low" frequencies (remember the scope is single-ended input).

Without connecting the alligator clip, the "emf loop" can be quite large since the ground loop passes through the o'scope power cord, through the lab bench electrical power wiring, back through the function generator power cord, through the function generator test leads then back to the circuit.

At high frequencies (say roughly > 100 MHz), this emf loop can cause the measured voltage to change as the leads are moved around! Not desirable.

With the alligator clip attached, the “emf loop” is greatly reduced in size.