PE 13.5 Evaluate the directivity of an antenna with normalized radiation intensity

$$U(\theta,\phi) = \begin{cases} \sin\theta & 0 \le \theta \le \pi/2,\ 0 \le \phi \le 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

(13.40) $$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} (\sin\theta)\sin\theta\, d\theta\, d\phi = 2\pi \int_{\theta=0}^{\pi/2} \sin^2\theta\, d\theta$$

$$= 2\pi\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]\Bigg|_{\theta=0}^{\pi/2} = 2\pi\left[\left(\pi/4 - 0\right) - (0-0)\right]$$

$$\underline{P_{rad} = \pi^2/2}$$

(13.42) $$D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{rad}} = \frac{4\pi \sin\theta}{\pi^2/2} \qquad \begin{matrix} 0 \le \theta \le \pi/2 \\ 0 \le \phi \le 2\pi \end{matrix}$$

$$\underline{D(\theta,\phi) = \frac{8\sin\theta}{\pi} \qquad \begin{matrix} 0 \le \theta \le \pi/2 \\ 0 \le \phi \le 2\pi \end{matrix}}$$

$$\underline{D_{max} = 8/\pi = 2.5465 \quad (@\ \theta = \pi/2)}$$