13.30 An antenna has a far-field electric field given by

$$\mathbf{E}_s = \frac{I_o}{r} e^{-j\beta r} \sin\theta \, \mathbf{a}_\theta$$

where I_o is the maximum input current. Determine the value of I_o to radiate a power of 50 mW.

From Chapter 10

$$\bar{P}_{ave} = \tfrac{1}{2} \mathrm{Re}\{\bar{E}_s \times \bar{H}_s^*\} = \hat{a}_r \frac{1}{2} \frac{|\bar{E}_s|^2}{\eta}$$

$$= \hat{a}_r \frac{I_o^2 \sin^2\theta}{2\eta r^2}$$

Per (13.40), $P_{rad} = \oint\oint_S \bar{P}_{ave} \cdot d\bar{S}_r = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{I_o^2 \sin^2\theta}{2\eta r^2} r^2 \sin\theta \, d\theta \, d\phi$

$$P_{rad} = \frac{I_o^2}{2\eta} \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta$$

$$= \frac{I_o^2}{2\eta} \left(\phi\Big|_0^{2\pi}\right)\left[\frac{-1}{3}\cos\theta(\sin^2\theta + 2)\right]\Big|_{\theta=0}^{\pi}$$

$$= \frac{I_o^2}{2\eta}(2\pi - 0)\left[\left(\frac{-1}{3}\cos\pi(\sin^2\pi + 2)\right) - \left(-\tfrac{1}{3}\cos 0(\sin^2 0 + 2)\right)\right]$$

$$P_{rad} = \frac{I_o^2}{2\eta}(2\pi)(4/3) = \frac{4\pi I_o^2}{3\eta}$$

$$\hookrightarrow I_o^2 = \frac{3\eta P_{rad}}{4\pi} = \frac{3(376.7303)(50\times10^{-3})}{4\pi}$$

$$I_o^2 = 4.4968867$$

$$\underline{\underline{I_o = 2.1206 \text{ A}}}$$