

13.28 For each of the following radiation intensities, calculate the directive gain and directivity.

(a) $U(\theta, \phi) = 10 \sin \theta \sin^2 \phi \quad 0 < \theta < \pi, 0 < \phi < 2\pi$

Per (13.40), the power radiated is

$$\begin{aligned} P_{\text{rad}} &= \iint_S U(\theta, \phi) \sin \theta \, d\theta \, d\phi \\ &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} 10 \sin^2 \theta \sin^2 \phi \, d\theta \, d\phi \\ &= 10 \int_{\phi=0}^{2\pi} \sin^2 \phi \, d\phi \int_{\theta=0}^{\pi} \sin^2 \theta \, d\theta = 10 \left(\frac{\phi}{2} - \frac{\sin 2\phi}{4} \right) \Big|_{\phi=0}^{2\pi} \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\theta=0}^{\pi} \\ &= 10 \left[\left(\frac{2\pi}{2} - \frac{\sin 4\pi}{4} \right) - \left(\frac{0}{2} - \frac{\sin 0}{4} \right) \right] \left[\left(\frac{\pi}{2} - \frac{\sin 2\pi}{4} \right) - \left(\frac{0}{2} - \frac{\sin 0}{4} \right) \right] \\ &= 10 (\pi) \left(\frac{\pi}{2} \right) = \underline{5\pi^2} \end{aligned}$$

Per (13.42), the directive gain is

$$G_d(\theta, \phi) = D(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{\text{rad}}} = \frac{4\pi (10) \sin \theta \sin^2 \phi}{5\pi^2}$$

$$\underline{D(\theta, \phi) = 2.54648 \sin \theta \sin^2 \phi \quad \begin{array}{l} 0 < \theta < \pi \\ 0 < \phi < 2\pi \end{array}}$$

$$D_{\text{max}} = 2.54648 = 10 / 09_{10} \, 2.5465$$

$$\underline{D_{\text{max}} = 2.5465 = 4.0594 \text{ dBi}}$$