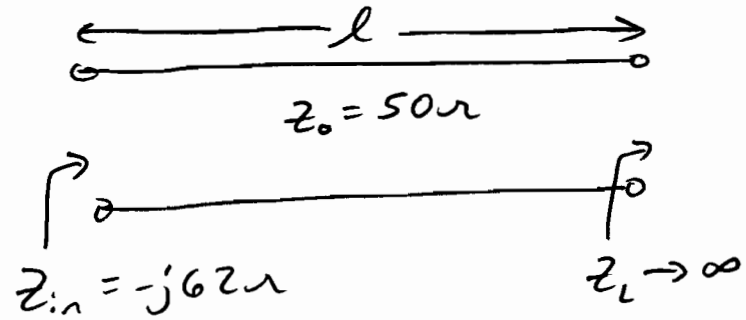
11.27 A 50 Ω transmission line of length ℓ is open-circuited. If the input impedance is $-j62\ \Omega$, determine ℓ in terms of λ.



Normalize impedances and plot on Smith Chart

$$z_L = \frac{\infty}{50} \to \infty$$

$$z_{in} = \frac{-j62}{50} = -j1.24\ \Omega/\Omega$$

looking at the "WAVELENGTHS TOWARD GENERATOR" scale

$$\ell = 0.3575\lambda - 0.25\lambda$$

↑ z_{in} location ↑ $z_L = z_{oc}$ location

$$\underline{\ell = 0.1075\lambda + n\,\lambda/2}$$

↑ can always add half wavelengths

Analytic Check

$$\Gamma_{in} = \frac{z_{in} - z_0}{z_{in} + z_0} = \frac{-j62 - 50}{-j62 + 50} = 1\angle -77.77^\circ = 1\angle -1.3573\,\text{rad}$$

Per Smith Chart

$$\Gamma_{in} = \Gamma_L e^{-j2\beta\ell} = (1)\, e^{-j2\left(\frac{2\pi}{\lambda}\right)(0.1075\lambda)} = 1e^{-j1.3509} = 1\angle -1.3509\,\text{rad}$$

↑ open ckt

↕ fairly close!

Simple Smith Chart

Problem 11.27

$Z_0 = 50\Omega$