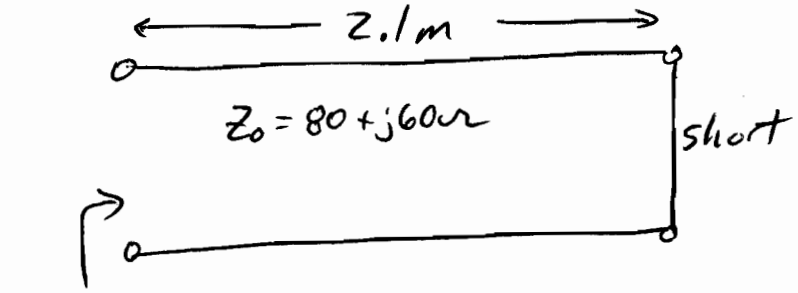
11.22 A lossy transmission line of length 2.1 m has characteristic impedance of $80 + j60\ \Omega$. When the line is short-circuited, the input impedance is $30 - j12\ \Omega$. (a) Determine α and β. (b) Find the input impedance when the short circuit is replaced by $Z_L = 40 + j30\ \Omega$.



$Z_{in} = 30 - j12\ \Omega$

a) Per (11.33), for a short circuit stub ($Z_L = 0$)

$$Z_{in,sc} = Z_0 \tanh(\gamma \ell)$$

$$30 - j12 = (80 + j60)\tanh(\gamma(2.1))$$

$$2.1\gamma = \tanh^{-1}\left(\frac{30 - j12}{80 + j60}\right) = 0.1571 - j0.276176$$

$$\gamma = 0.07480544 - j0.1315125\ m^{-1}$$

$$\alpha = 0.074805\ Np/m \quad \& \quad \beta = 0.1315125\ rad/m$$

b) Use (11.33) again

$$Z_{in} = Z_0\left[\frac{Z_L + Z_0 \tanh\gamma\ell}{Z_0 + Z_L \tanh\gamma\ell}\right]$$

$$= (80 + j60)\left[\frac{(40 + j30) + (80 + j60)\left(\frac{30 - j12}{80 + j60}\right)}{(80 + j60) + (40 + j30)\left(\frac{30 - j12}{80 + j60}\right)}\right]$$

$$Z_{in} = 61.4655 + j24.4301\ \Omega$$