11.11 A telephone line operating at 1 kHz has $R = 6.8\ \Omega/\text{mi}$, $L = 3.4\ \text{mH/mi}$, $C = 8.4\ \text{nF/mi}$, and $G = 0.42\ \mu\text{S/mi}$. Find (a) Z and γ, (b) phase velocity, (c) wavelength.

a) $$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{6.8 + j2\pi 1000(3.4\times10^{-3})}{0.42\times10^{-6} + j2\pi 1000(8.4\times10^{-9})}}$$

$$Z_0 = 644.407 - j97.462\ \Omega = 651.74\angle -8.60^\circ\ \Omega$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(6.8+j2000\pi(3.4\times10^{-3}))(0.42\times10^{-6}+j2000\pi(8.4\times10^{-9}))}$$

$$= 0.00541459 + j0.03397\ \frac{1}{\text{miles}}\left(\frac{1\ \text{mile}}{1609.344\ \text{m}}\right)$$

$$\gamma = 3.3645\times10^{-6} + j21.108\times10^{-6}\ \frac{1}{\text{m}}$$

b) $$u = \frac{\omega}{\beta} = \frac{2\pi(1000)}{21.10802\times10^{-6}} = 2.97668\times10^{8}\ \text{m/s}$$

$$= \frac{2000\pi}{0.03397} = 184{,}962.66\ \text{miles/s}\quad \text{or}$$

c) $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{21.10802\times10^{-6}} = 297{,}668.15\ \text{m} = 297.668\ \text{km}$$

$$= \frac{2\pi}{0.03397} = 184.963\ \text{miles}\quad \text{or}$$