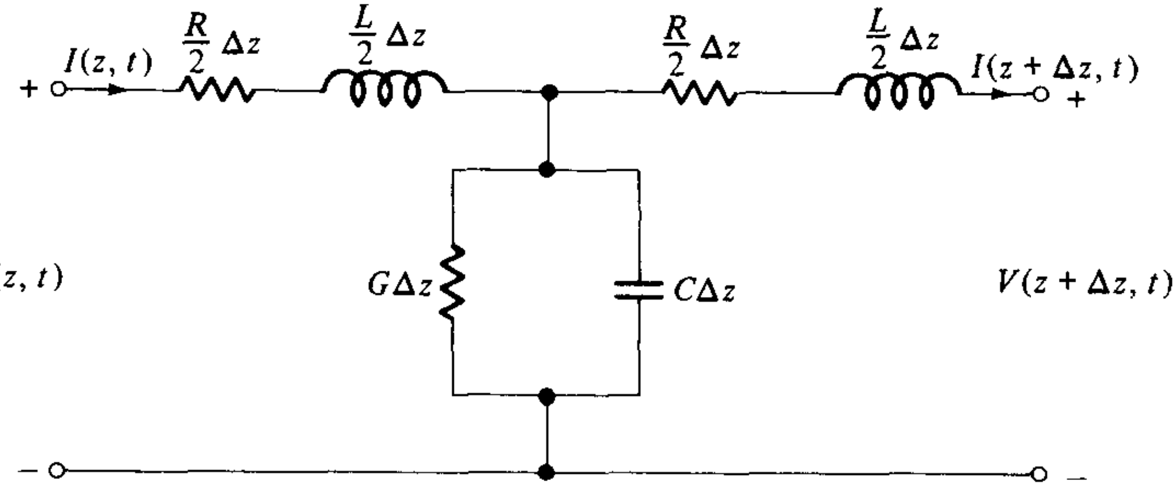
11.6 In Section 11.3, it was mentioned that the equivalent circuit of Figure 11.5 is not the only possible one. Show that eqs. (11.4) and (11.6) would remain the same if the Π-type and T-type equivalent circuits shown in Figure 11.47 were used.



(b)

FIGURE 11.47 Equivalent circuits for Problem 11.6: **(b)** *T*-type.

Apply KVL to outer loop

$$-V(z,t) + I(z,t)\frac{R}{2}\Delta z + \frac{L}{2}\Delta z\frac{\partial I(z,t)}{\partial t} + I(z+\Delta z,t)\,R/2\,\Delta z + \frac{L}{2}\Delta z\frac{\partial I(z+\Delta z,t)}{\partial t} + V(z+\Delta z,t) = 0$$

$$-\left[\frac{V(z+\Delta z,t)-V(z,t)}{\Delta z}\right] = R/2\left[I(z,t)+I(z+\Delta z,t)\right] + \frac{L}{2}\left[\frac{\partial\left(I(z,t)+I(z+\Delta z,t)\right)}{\partial t}\right]$$

letting $\Delta z \to 0$ $\quad I(z+\Delta z,t) \to I(z,t)$, so

$$-\frac{\partial V(z,t)}{\partial z} = R/2\left[2I(z,t)\right] + L/2\,\frac{\partial\left(2I(z,t)\right)}{\partial t}$$

$$\therefore \boxed{-\frac{\partial V(z,t)}{\partial z} = R\,I(z,t) + L\frac{\partial I(z,t)}{\partial t}} \quad (11.4)$$

Apply KCL at top center node

$$-I(z,t) + I(z+\Delta z,t) + G\Delta z\, V\left(z+\tfrac{\Delta z}{2},t\right) + C\Delta z \frac{\partial V\left(z+\frac{\Delta z}{2},t\right)}{\partial t} = 0$$

$$-\left[\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z}\right] = G V\left(z+\tfrac{\Delta z}{2},t\right) + C\frac{\partial V\left(z+\frac{\Delta z}{2},t\right)}{\partial t}$$

letting $\Delta z \to 0$ $\quad V\left(z+\frac{\Delta z}{2},t\right) \to V(z,t)$ +

$$\therefore \boxed{-\frac{\partial I(z,t)}{\partial z} = G V(z,t) + C\frac{\partial V(z,t)}{\partial t}} \qquad (11.6)$$