A plane wave traveling in the +*y*-direction in a lossy medium ($\varepsilon_r = 4$, $\mu_r = 1$, $\sigma = 10^{-2}$ S/m) has $\bar{\mathcal{E}} = 30\cos(10^9 \pi t + \pi/4)\,\hat{a}_z$ V/m at y = 0. Find:

a) $\bar{\mathcal{E}}$ at $y = 1m$, $t = 2ns$

(10.18) $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \sqrt{j10^9\pi(4\pi \times 10^{-7})(10^{-2} + j10^9\pi(4)8.854\times10^{-12})}$

$\gamma = 0.9408881 + j\,20.9793378\ m^{-1}$

↑ α ↑ β

for +y-dir, the phase term is $-\beta y$, and adapting (10.29) yields

$e^{-\alpha y}$ $-\beta y$

$\bar{\mathcal{E}}(y,t) = 30e^{-0.94y}\cos(10^9\pi t - 20.979y + \pi/4)\,\hat{a}_z$ V/m

$\bar{\mathcal{E}}(1, 2\times10^{-9}) = 30e^{-0.940888}\cos(2\pi - 20.979337 + \pi/4)\,\hat{a}_z$ V/m

$\bar{\mathcal{E}}(y{=}1m, 2ns) = 2.62835\,\hat{a}_z$ V/m

b) The distance traveled by the wave to have a phase shift of 10°.

phase shift $= \beta y = 20.979337\,y = 10°(\pi/180°)$

$y = 8.32mm$

c) The distance traveled by the wave to have its amplitude reduced by 40%. wave left is 100% − 40% = 60%

$$\Rightarrow e^{-\alpha y} = 0.6 = e^{-0.9408881 y}$$

$$y = \frac{1}{-0.9408881} \ln(0.6)$$

$$y = 0.5429 \text{ m}$$

d) $\bar{H}$ at $y = 2m$, $t = 2ns$ $\qquad \hat{a}_z \times \hat{a}_H = \hat{a}_y \Rightarrow \hat{a}_H = \hat{a}_x$

$$\bar{H} = \frac{E_0 e^{-\alpha y}}{|\eta|} \cos(\omega t - \beta y + \pi/4 - \theta_\eta)\, \hat{a}_x \ \text{A/m}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\,10^9\pi(4\pi\times10^{-7})}{10^{-2} + j\,10^9\pi(4)\,8.854\times10^{-12}}}$$

$$= 187.98866 \angle 2.5678989^\circ\ \Omega$$

$$\bar{H} = \frac{30}{187.99} e^{-0.940888 y} \cos(10^9\pi t - 20.97934 y + \pi/4 - 2.568^\circ)\, \hat{a}_x \ \frac{A}{m}$$

convert to rad.

$$\bar{H}(2, 2\times10^{-9}) = \frac{30}{187.99} e^{-0.940888(2)} \cos(2\pi - 20.979(2) + \pi/4 - 2.568^\circ)\hat{a}_x$$

$$\bar{H}(y = 2m, 2ns) = -0.0226\, \hat{a}_x \ \text{A/m} = -22.6\, \hat{a}_x \ \text{mA/m}$$