10.58 A uniform plane wave in air is normally incident on an infinite lossless dielectric material occupying $z > 0$ and having $\varepsilon = 3\varepsilon_o$ and $\mu = \mu_o$. If the incident wave is $\mathbf{E}_i = 10\cos(\omega t - z)\mathbf{a}_y$ V/m, find

(a) λ and ω of the wave in air and the transmitted wave in the dielectric medium

(b) The incident $\mathbf{H}_i$ field

(c) Γ and τ

(d) The total electric field and the time-average power density in both regions

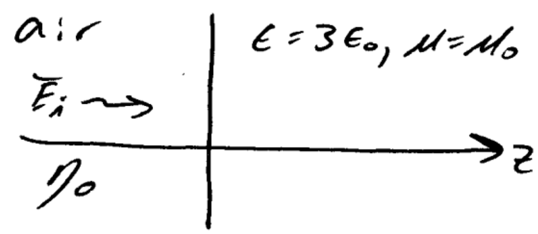
a) From $\bar{E}_i$, note that $\beta = 1 \frac{rad}{m}$. Per (10.8),

$$\beta = 1 \frac{rad}{m} = \frac{2\pi}{\lambda} \Rightarrow \underline{\underline{\lambda_{air} = 2\pi \text{ m} = 6.2832 \text{ m}}}$$

"In air" $\Rightarrow u = c = 2.9979 \times 10^8$ m/s

Per (10.7b), $\beta = 1 \frac{rad}{m} = \frac{\omega}{u} = \frac{\omega}{2.9979\times10^8} \Rightarrow \underline{\underline{\omega = 2.9979\times10^8 \frac{rad}{s}}}$

(for both)



Per (10.43b), $u_2 = \frac{1}{\sqrt{\mu_2 \epsilon_2}} = \frac{1}{\sqrt{\mu_o 3\epsilon_o}} = \frac{c}{\sqrt{3}} = 1.73084\times10^8$ m/s

$$\lambda_2 = \frac{u_2}{f} = \frac{1.73084\times10^8}{2.9979\times10^8/2\pi} \Rightarrow \underline{\underline{\lambda_2 = 3.6276 \text{ m}}}$$

b) Use (10.106), $\bar{H}_s = \frac{\hat{a}_k \times \bar{E}_s}{\eta} \Rightarrow \bar{H}_{s,i} = \frac{\hat{a}_z \times 10 e^{-jz}\hat{a}_y}{376.7303\ \Omega}$

$\bar{H}_{s,i} = -\hat{a}_x\, 0.026544\, e^{-jz}$ A/m

$\bar{\mathcal{H}}_i = \mathrm{Re}\{\bar{H}_{s,i} e^{j\omega t}\}$

$\bar{\mathcal{H}}_i = -\hat{a}_x\, 26.544 \cos(\omega t - z)$ mA/m

$$\underline{\underline{\bar{\mathcal{H}}_i = -\hat{a}_x\, 26.544 \cos(2.9979\times10^8 t - z) \text{ mA/m}}}$$

c) Per (10.91a), $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ and (10.92a) $\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}$

where, per (10.44), $\eta = \sqrt{\mu/\epsilon}$

$$\underline{\eta_1 = \eta_0 = \sqrt{\mu_0/\epsilon_0} = 376.7303135\,\Omega}$$

$$\underline{\eta_2 = \sqrt{\frac{\mu_0}{3\epsilon_0}} = \eta_0/\sqrt{3} = 217.50535\,\Omega}$$

$$\Gamma = \frac{217.5 - 376.73}{217.5 + 376.73} \Rightarrow \underline{\underline{\Gamma = -0.26795}}$$

$$\tau = \frac{2(217.5)}{217.5 + 376.73} \Rightarrow \underline{\underline{\tau = 0.73205}}$$

d) Per (10.91b), $E_{r0} = \Gamma E_{i0} = -0.26795(10) = -2.6795$ V/m
where $\bar{E}_r$ propagates in $-z$-direction

$$\bar{\mathcal{E}}_r = -2.6795 \cos(\omega t + z)\,\hat{a}_y \text{ V/m}$$

$$\bar{\mathcal{E}}_{Tot,air} = \bar{\mathcal{E}}_i + \bar{\mathcal{E}}_r = 10\cos(\omega t - z)\,\hat{a}_y - 2.6795\cos(\omega t + z)\,\hat{a}_y$$

$$\underline{\underline{\bar{\mathcal{E}}_{TOT,air} = \hat{a}_y\left[10\cos(2.9979\times10^8 t - z) - 2.6795\cos(2.9979\times10^8 t + z)\right]\frac{V}{m}}}$$

Per (10.92b), $E_{t0} = \tau E_{i0} = 0.73205(10) = 7.3205$ V/m
where $\bar{E}_t$ still propagates in $+z$-dir. w/ $\beta_2 = \frac{\omega}{u_2} = 1.73205\ \frac{rad}{m}$

$$\underline{\underline{\bar{\mathcal{E}}_{TOT,2} = \bar{\mathcal{E}}_t = \hat{a}_y\, 7.3205\cos(2.9979\times10^8 t - 1.73205 z) \text{ V/m}}}$$

Per (10.78), $\bar{P}_{ave} = \frac{1}{2}\mathrm{Re}\{\bar{E}_s \times \bar{H}_s^*\}$ & (10.106) $\bar{H}_s = \dfrac{\hat{a}_k \times \bar{E}_s}{\eta}$

$\hookrightarrow \bar{P}_{ave} = \hat{a}_k \dfrac{|E_s|^2}{2\eta}$ $\quad \bar{P}_{ave,air} = \hat{a}_z \dfrac{10^2 - 2.6795^2}{2(373.73)} = \underline{\underline{\hat{a}_z\, 0.1232\ \frac{W}{m^2}}}$

$$\bar{P}_{ave,2} = \hat{a}_z \frac{7.3205^2}{2(376.73/\sqrt{3})} = \underline{\underline{\hat{a}_z\, 0.1232\ \text{W/m}^2}}$$