10.41 The electric field intensity in a dielectric medium ($\mu = \mu_o$, $\varepsilon = \varepsilon_o \varepsilon_r$) is given by

$$\mathbf{E} = 150 \cos(10^9 t + 8x)\mathbf{a}_z \text{ V/m}$$

Calculate

(a) The dielectric constant ε_r

(b) The intrinsic impedance

(c) The velocity of propagation

(d) The magnetic field intensity

(e) The Poynting vector $\mathcal{P}$

a) Given no $e^{\alpha x}$ term, we can assume $\sigma = 0$. For a lossless dielectric (10.43a) $\beta = \omega\sqrt{\mu\varepsilon}$. From the expression for $\bar{E}$, $\omega = 10^9 \frac{rad}{s}$ and $\beta = 8 \ rad/m$

$$8 = 10^9 \sqrt{4\pi \times 10^{-7} (\varepsilon_r)\, 8.8541878 \times 10^{-12}}$$

$\hookrightarrow$ $\underline{\underline{\varepsilon_r = 5.752}}$

b) Per (10.44), $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{4\pi \times 10^{-7}}{5.752(8.8542 \times 10^{-12})}} \Rightarrow \underline{\underline{\eta = 157.08\ \Omega}}$

c) Per (10.43b), $u = \frac{\omega}{\beta} = \frac{10^9}{8} \Rightarrow \underline{\underline{u = 1.25 \times 10^8\ m/s}}$

d) Per (10.106), $\bar{H} = \frac{\hat{a}_k \times \bar{E}}{\eta} = \frac{-\hat{a}_x \times \hat{a}_z\, 150\, e^{j8x}}{157.08}$

$$= \hat{a}_y\, 0.954927\, e^{j8x}\ A/m$$

$\mathcal{H} = \mathrm{Re}\{\bar{H} e^{j10^9 t}\} \Rightarrow \underline{\underline{\mathcal{H} = \hat{a}_y\, 0.95493 \cos(10^9 t + 8x)\ A/m}}$

e) Per (10.75), $\mathcal{P} = \mathcal{E} \times \mathcal{H} = \hat{a}_z 150 \cos(10^9 t + 8x) \times \hat{a}_y 0.95493 \cos(10^9 t + 8x)$

$$\underline{\underline{\mathcal{P} = -\hat{a}_x\, 143.239 \cos^2(10^9 t + 8x)\ \ W/m^2}}$$