10.37 The electric field intensity of a uniform plane wave in free space is given by

$$\mathbf{E} = 40\cos(\omega t - \beta z)\mathbf{a}_x + 60\sin(\omega t - \beta z)\mathbf{a}_y \text{ V/m}$$

(a) What is the wave polarization?

(b) Determine the magnetic field intensity.

- For part a), plot the polarization ellipse with axes selected so that the wave propagates into the page. Determine the sense, AR, & tilt angle τ with respect to the vertical axis.

a) $\bar{E} = 40\cos(\omega t - \beta z)\,\hat{a}_x + 60\cos(\omega t - \beta z - 90°)\,\hat{a}_y$ V/m

2 components of unequal magnitude that are 90° out-of-phase

→ can't be linear due to 90° phase diff.

→ can't be circular due to unequal magnitudes

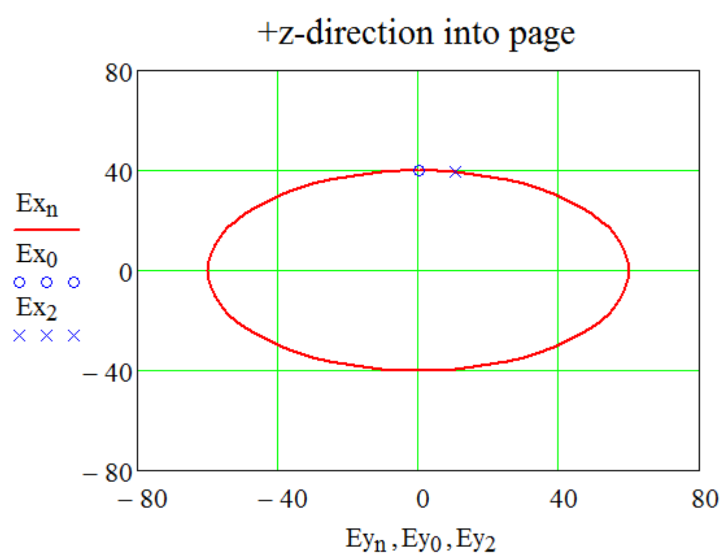
⇒ Elliptical Polarization

1) Arbitrarily chose the z = 0 plane to plot the polarization ellipse.

2) Do the plot for one period of time, i.e., 0 < ωt < 2π.

$n := 0..72$ $\omega t_n := n\cdot\frac{2\pi}{72}$ $\beta z := 0$

$Ex_n := 40\cdot\cos(\omega t_n - \beta z)$ V/m $Ey_n := 60\cdot\cos\left(\omega t_n - \beta z - \frac{\pi}{2}\right)$ V/m



From plot, the **sense** of the polarization ellipse is **RH/righthand** or **CW/clockwise**.

From plot, the axial ratio AR = 2*60/2*40 = 120/80 ⇒ **AR = 1.5**.

From plot, the tilt angle τ with respect to the vertical/E_x axis is **τ = 90°**.

b) $\bar{H} = \dfrac{\hat{a}_k \times \bar{E}}{\eta}$ where $\hat{a}_k = +\hat{a}_z$ from "$-\beta z$" term + $\eta = \eta_0 = 376.73\,\Omega$ (free space)

$$= \hat{a}_z \times \left[\frac{40}{376.73} e^{-j\beta z} \hat{a}_x + \frac{60}{376.73} e^{-j(\beta z + 90^\circ)} \hat{a}_y \right]$$

$$= \hat{a}_y\, 0.106177\, e^{-j\beta z} - \hat{a}_x\, 0.159265\, e^{-j(\beta z + 90^\circ)}$$

$$\bar{\mathcal{H}} = \mathrm{Re}\{\bar{H} e^{j\omega t}\}$$

$$\bar{\mathcal{H}} = -\hat{a}_x\, 0.1593 \cos(\omega t - \beta z - 90^\circ) + \hat{a}_y\, 0.1062 \cos(\omega t - \beta z) \text{ A/m}$$

$$= -\hat{a}_x\, 0.1593 \sin(\omega t - \beta z) + \hat{a}_y\, 0.1062 \cos(\omega t - \beta z) \text{ A/m}$$