10.12 In a certain medium with $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$, ($\omega$)

$$\mathbf{H} = 12e^{-0.1y}\sin(\pi \times 10^8 t - \beta y)\mathbf{a}_x \text{ A/m}$$

Find (a) the wave period T, (b) the wavelength λ, (c) the electric field $\mathbf{E}$, (d) the phase difference between $\mathbf{E}$ and $\mathbf{H}$.

a) Per (10.7c), $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi \times 10^8} = \underline{20\,ns}$

b) To get λ, we need the phase constant β. From the expression for $\bar{H}$, we see that $\alpha = 0.1\ Np/m$

Per (10.18) $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) = j\omega\mu\sigma - \omega^2\mu\varepsilon$

Per (10.20) $\gamma = \alpha + j\beta \Rightarrow \gamma^2 = \alpha^2 - \beta^2 + j2\alpha\beta$

Equate the real parts of γ^2

$$-\omega^2\mu\varepsilon = -(\pi \times 10^8)^2 4\pi \times 10^{-7}(4)(8.8541878 \times 10^{-12}) = 0.1^2 - \beta^2$$

$$\hookrightarrow \beta = \sqrt{0.1^2 + 4.39256635} = 2.098229337\ rad/m$$

Per (10.8), $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2.09823} = \underline{2.9945\,m}$

c) Equate real parts of γ^2: $j\omega\mu\sigma = j2\alpha\beta$

$$\sigma = \frac{j2(0.1)(2.098229337)}{j\pi \times 10^8 (4\pi \times 10^{-7})} = 0.001062975\ S/m$$

Calculate intrinsic impedance η using (10.32)

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\pi \times 10^8 (4\pi \times 10^{-7})}{0.001063 + j\pi \times 10^8 (4) 8.854 \times 10^{-12}}}$$

$$= 187.93778886 \angle 2.728608^\circ\ \Omega$$

c) cont. use $\bar{E}_s = \eta(\bar{H}_s \times \hat{a}_k)$

$\bar{E}_s = (87.939 \angle 2.729^\circ)\left[12 e^{-0.1y} e^{-j\beta y} \hat{a}_x \times \hat{a}_y\right)$ ← from "$-j\beta y$" term

$= 2255.2535\, e^{-0.1y} e^{j2.729^\circ} e^{-j\beta y} \hat{a}_z \qquad \beta = 2.098 \frac{rad}{m}$

$\bar{\mathcal{E}} = 2255.25 e^{-0.1y} \cos(\pi \times 10^8 t - 2.098z + 2.729^\circ)\, \hat{a}_z \; V/m$

d) Comparing $\mathcal{H}$ & $\mathcal{E}$ $\Rightarrow$ $\Delta phase = 2.729^\circ$