10.6 At 50 MHz, a lossy dielectric material is characterized by $\varepsilon = 3.6\varepsilon_o$, $\mu = 2.1\mu_o$, and $\sigma = 0.08$ S/m. If $\mathbf{E}_s = 6e^{-\gamma x}\,\mathbf{a}_z$ V/m, compute (a) γ, (b) λ, (c) u, (d) η, (e) $\mathbf{H}_s$.

- Also, (f) loss tangent.

a) Per (10.18), $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$$\gamma = \sqrt{j2\pi(50\times10^6)(2.1)4\pi\times10^{-7}\left(0.08 + j2\pi(50\times10^6)3.6(8.8541878\times10^{-12})\right)}$$

$$\gamma = 5.41015 + j6.12957 \ m^{-1}$$

(α = 5.41015, β = 6.12957)

b) Per (10.8), $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6.12957} = 1.02506$ m

c) Per (10.76), $u = \frac{\omega}{\beta} = \frac{2\pi\times50\times10^6}{6.12957} = 5.1253\times10^7$ m/s

d) Per (10.32), $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j2\pi(50\times10^6)2.1(4\pi\times10^{-7})}{0.08 + j2\pi(50\times10^6)3.6(8.8541878\times10^{-12})}}$

$$\eta = 101.40434\,\angle 41.43261^\circ\ \Omega$$

e) $\bar{H}_s = \hat{a}_{ic} \times \frac{\bar{E}_s}{\eta}$ (10.106)

$$= \hat{a}_x \times \frac{6e^{-\gamma x}\hat{a}_z}{101.4\,\angle 41.43^\circ}$$

$$= -\hat{a}_y\, 0.059169\,\angle -41.43^\circ e^{-\gamma x}$$

$$= -\hat{a}_y\, 59.169\, e^{-j41.433^\circ} e^{-5.41x} e^{-j6.1296x} \ mA/m$$

$$\bar{H}_s = -\hat{a}_y\, 59.169\, e^{-5.41x} e^{-j(6.1296x + 41.433^\circ)} \ mA/m$$

f) (10.37) $\tan\theta = \frac{\sigma}{\omega\varepsilon} = \frac{0.08}{2\pi(50\times10^6)3.6(8.8541878\times10^{-12})} = 7.989$