Write the following vector phasor quantities in time domain form. The frequency in each case is 30 MHz.

(a) $\bar{E} = -\hat{a}_x j30 - \hat{a}_y 10/j$ V/m

(b) $\bar{H} = \hat{a}_z 10e^{j4\pi/5}$ A/m

(c) $\rho = j4e^{-j\pi/3}$ C/m

(d) $\bar{B} = \hat{a}_x j2e^{(-j4\pi/3)z}e^{-20z}$ T

(e) $\bar{A} \times \bar{B}^*$ where $\bar{A} = \hat{a}_y j3e^{-j2x} + \hat{a}_z 2e^{-3x}$ and $\bar{B} = -\hat{a}_x je^{-jx} - \hat{a}_z(1+j)e^{-jx}$

$\omega = 2\pi f = 2\pi \times 30\times10^6 = 188.5\times10^6$ rad/s

From Euler's Identity, $e^{\pm jA} = \cos A \pm j\sin A$, $j = e^{j\pi/2}$ & $-j = e^{-j\pi/2}$

a) $\bar{E} = -j30\,\hat{a}_x - \frac{10}{j}\hat{a}_y = 30e^{-j\pi/2}\hat{a}_x + 10e^{+j\pi/2}\hat{a}_y$ V/m

$\bar{\mathcal{E}} = \mathrm{Re}\{\bar{E}e^{j\omega t}\} = \mathrm{Re}\{30e^{-j\pi/2}e^{j\omega t}\hat{a}_x + 10e^{j\pi/2}e^{j\omega t}\hat{a}_y\}$

$= \mathrm{Re}\{[30\cos(\omega t - \pi/2) + j30\sin(\omega t - \pi/2)]\hat{a}_x$
$+ [10\cos(\omega t + \pi/2) + j10\sin(\omega t + \pi/2)]\hat{a}_y\}$

$\bar{\mathcal{E}} = 30\cos(\omega t - \pi/2)\hat{a}_x + 10\cos(\omega t + \pi/2)\hat{a}_y$

$\bar{\mathcal{E}} = 30\cos(188.5\times10^6 t - \pi/2)\hat{a}_x + 10\cos(188.5\times10^6 t + \pi/2)\hat{a}_y$ (V/m)

b) $\bar{\mathcal{H}} = \mathrm{Re}\{\bar{H}e^{j\omega t}\} = \mathrm{Re}\{\hat{a}_z 10e^{j4\pi/5}e^{j\omega t}\}$

$= \hat{a}_z 10\,\mathrm{Re}\{\cos(\omega t + 4\pi/5) + j\sin(\omega t + 4\pi/5)\}$

$\bar{\mathcal{H}} = 10\cos(188.5\times10^6 t + \frac{4\pi}{5})\hat{a}_z$ (A/m)

c) $\rho = j4e^{-j\pi/3} = 4e^{j\pi/2}e^{-j\pi/3} = 4e^{j\pi/6}$

$\rho(t) = \mathrm{Re}\{4e^{j\pi/6}e^{j\omega t}\} = 4\,\mathrm{Re}\{\cos(\omega t + \pi/6) + j\sin(\omega t + \pi/6)\}$

$\rho(t) = 4\cos(188.5\times10^6 t + \pi/6)$ (C/m)

d) $\bar{B} = j2e^{-j\frac{4\pi}{3}z}e^{-20z}\hat{a}_x = 2e^{j\pi/2}e^{-j\frac{4\pi}{3}z}e^{-20z}\hat{a}_x$ T

$\bar{\mathcal{B}} = \mathcal{R}e\{2e^{-20z}e^{j\pi/2}e^{-j\frac{4\pi}{3}z}e^{j\omega t}\hat{a}_x\}$

$= \hat{a}_x 2e^{-20z}\mathcal{R}e\{\cos(\omega t - \frac{4\pi}{3}z + \pi/2) + j\sin(\text{ditto})\}$

$$\underline{\underline{\bar{\mathcal{B}} = \hat{a}\ \ 2e^{-20z}\cos(188.5\times10^6 t - \tfrac{4\pi}{3}z + \pi/2)\ \ (T)}}$$

e) $\bar{A}\times\bar{B}^* = (\hat{a}_y 3e^{j\pi/2}e^{-j2x} + \hat{a}_z 2e^{-3x}) \times (-\hat{a}_x e^{j\pi/2}e^{-jx} - \hat{a}_z(1+e^{j\pi/2})e^{-jx})^*$

$= (\hat{a}_y 3e^{j\pi/2}e^{-j2x} + \hat{a}_z 2e^{-3x}) \times (-\hat{a}_x e^{-j\pi/2}e^{jx} - \hat{a}_z(1+e^{-j\pi/2})e^{jx})$

$= +\hat{a}_z 3e^{j\pi/2}e^{-j2x}e^{-j\pi/2}e^{jx} - \hat{a}_x 3e^{j\pi/2}e^{-j2x}(1+e^{-j\pi/2})e^{jx}$

$- \hat{a}_y 2e^{-3x}e^{-j\pi/2}e^{jx} + 0$

$= \hat{a}_x[-3e^{j\pi/2}e^{-jx} - 3e^{-jx}] + \hat{a}_y[-2e^{-3x}e^{-j\pi/2}e^{jx}]$

$+ \hat{a}_z[3e^{-jx}]$ use $-1 = e^{j\pi}$

$\mathcal{R}e\{(\bar{A}\times\bar{B}^*)e^{j\omega t}\} = -3\hat{a}_x\mathcal{R}e\{e^{j(\omega t - x + \pi/2)} + e^{j(\omega t - x)}\}$

$+ 2e^{-3x}\hat{a}_y\mathcal{R}e\{e^{j(\omega t + x + \pi/2)}\}$

$+ 3\hat{a}_z\mathcal{R}e\{e^{j(\omega t - x)}\}$

$$\boxed{\begin{aligned}\mathcal{R}e\{(\bar{A}\times\bar{B}^*)e^{j\omega t}\} = {} & \hat{a}_x[-3\cos(188.5\times10^6 t - x + \pi/2) - 3\cos(188.5\times10^6 t - x)] \\ & + \hat{a}_y[2e^{-3x}\cos(188.5\times10^6 t + x + \pi/2)] \\ & + \hat{a}_z[3\cos(188.5\times10^6 t - x)]\end{aligned}}$$