The electric field $\bar{E}(x,t) = \hat{a}_y E_0 \cos(\beta x)\sin(\omega t)$ V/m exists in free space. Using the phasor form of this electric field, find the phasor form of the magnetic field. From this, determine the time domain form of the magnetic field.

Use Trig. Identity $\sin(A) = \cos(A - \pi/2)$

$$\bar{\mathcal{E}}(x,t) = \hat{a}_y E_0 \cos(\beta x)\cos(\omega t - \pi/2) \text{ V/m} \quad \text{Phasor}$$

$$\bar{E}(x) = \hat{a}_y E_0 \cos(\beta x) e^{-j\pi/2} = -\hat{a}_y j E_0 \cos(\beta x) \text{ V/m}$$

Using the phasor form of Faraday's Law, w/ $\mu = \mu_0$ (free space), we get

$$(10.13) \quad \bar{\nabla} \times \bar{E} = -j\omega\mu_0 \bar{H}$$

where $\bar{\nabla} \times \bar{E} = \left[\frac{\partial E_z}{\partial y}^{\to 0} - \frac{\partial E_y}{\partial z}^{\to 0}\right]\hat{a}_x + \left[\frac{\partial E_x}{\partial z}^{0} - \frac{\partial E_z}{\partial x}^{0}\right]\hat{a}_y$

$$+ \left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}^{\to 0}\right]\hat{a}_z$$

$$= -j E_0 \frac{\partial \cos(\beta x)}{\partial x}\hat{a}_z = +j E_0 \beta$$

So $\bar{H} = \dfrac{\bar{\nabla} \times \bar{E}}{-j\omega\mu_0} = \dfrac{j E_0 \beta \sin(\beta x)}{-j\omega\mu_0}\hat{a}_z$

$$\underline{\underline{\bar{H}(x) = -\hat{a}_z \frac{\beta E_0}{\omega\mu_0}\sin(\beta x) \text{ A/m}}}$$

$$\bar{\mathcal{H}}(x,t) = \mathrm{Re}\{\bar{H} e^{j\omega t}\} = \mathrm{Re}\left\{-\hat{a}_z \frac{\beta E_0}{\omega\mu_0}\sin(\beta x) e^{j\omega t}\right\}$$

$$\underline{\underline{\bar{\mathcal{H}}(x,t) = -\hat{a}_z \frac{\beta E_0}{\omega\mu_0}\sin(\beta x)\cos(\omega t) \text{ (A/m)}}}$$

Note: $\dfrac{\beta}{\omega\mu_0} = \dfrac{\omega/c}{\omega\mu_0} = \dfrac{1}{c\mu_0} = \dfrac{\sqrt{\mu_0\epsilon_0}}{\mu_0} = \sqrt{\dfrac{\epsilon_0}{\mu_0}} = \dfrac{1}{\eta_0}$