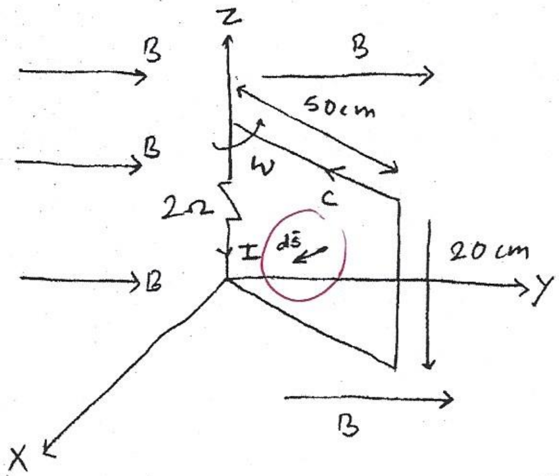
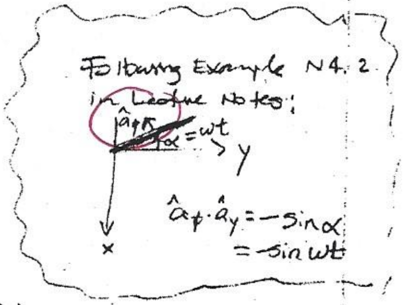
26 Jan 2018 EE 382 – Applied EM

[8pts] Prob. 1 Homework #3



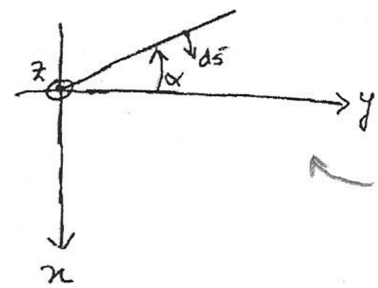


Given, $\omega = 10$ rad/s

$\vec{B} = \hat{a}_y\, 30$ mT

$ds = -\hat{a}_\phi\, d\rho\, dz$

Top view:



1pt

$$emf = -\frac{d}{dt}\int_s \vec{B}\cdot\vec{ds}$$

$$= -\frac{d}{dt}\int_0^{0.2}\int_0^{0.5}\left(\hat{a}_y\, 30\times10^{-3}\right)\cdot\left(-\hat{a}_\phi\, \partial\rho\, dz\right)$$ 1pt

$$= \frac{d}{dt}\int_0^{0.2}\int_0^{0.5} 30\times10^{-3}\,\underbrace{\hat{a}_y\cdot\hat{a}_\phi}_{\text{see above}}\,\partial\rho\,\partial z \quad = -\sin\omega t$$ 1pt

$$= -30\times10^{-3}\frac{d}{dt}\int_0^{0.2}\int_0^{0.5}\sin\omega t\,\partial\rho\,\partial z$$ 1pt

$$= -30\times10^{-3}\frac{d}{dt}\sin\omega t\int_0^{0.2}\rho\Big|_0^{0.5}\partial z$$ 1pt

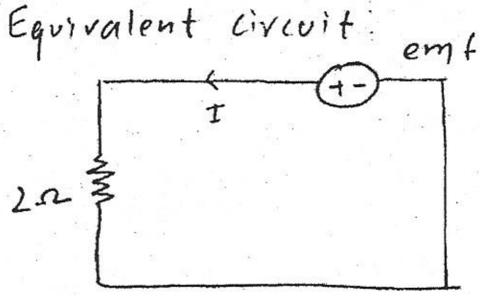
$= -30 \times 10^{-3} \dfrac{d}{dt} \sin \omega t \times 0.5 \times 0.2$ ⇐ OK!

$= -30 \times 10^{-3} \times 0.5 \times 0.2 \times (\cos \omega t) \times \omega$

$= -30 \times 10^{-3} \times 0.5 \times 0.2 \times 10 \times \cos 10t$ ← 1pt

$= -0.03 \cos 10t$ V ← 1pt

Equivalent circuit:



$I = \dfrac{emf}{2}$

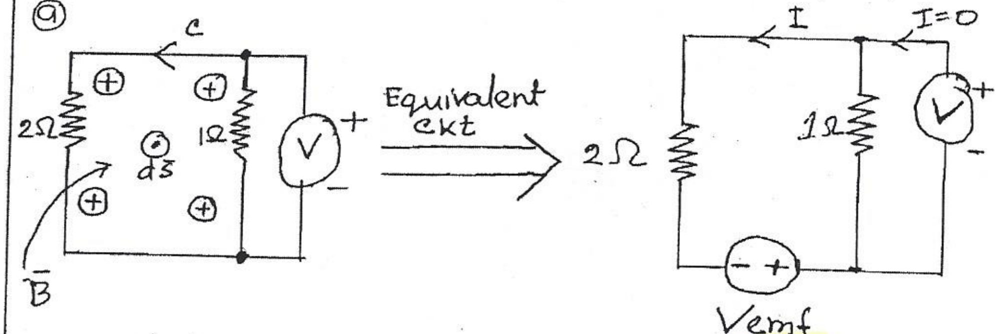
$= \dfrac{-0.03 \cos 10t}{2}$ ← 1pt

$= -0.015 \cos 10t$ A

[9 pts] Prob. 2

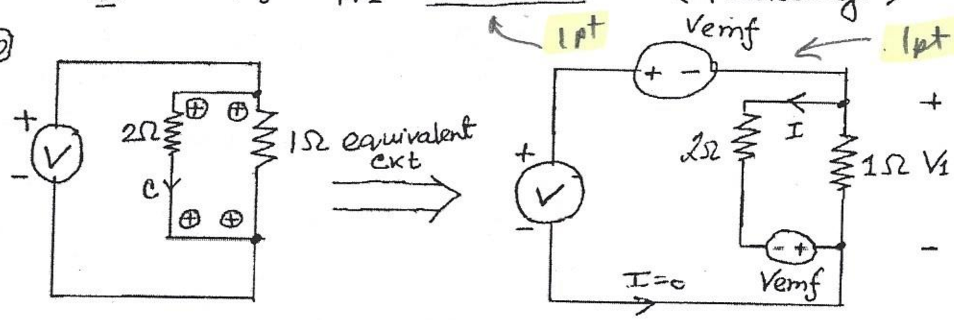
$A = 1\ m^2$
$B = 0.1\,t\ \frac{wb}{m^2}$

ⓐ



• $\Psi_m = -BA = -0.1t$ wb ← 1pt
• $V_{emf} = -\frac{\partial \Psi_m}{\partial t} = 0.1\ V$ ← 1pt
$\Rightarrow \underline{V} = -V_{emf} \times \frac{1}{1+2} = \underline{\underline{-33.3\ mV}}$ (Voltmeter reading) ← 1pt

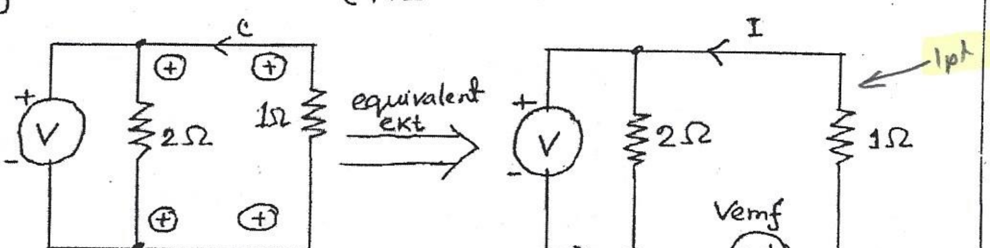
ⓑ



1pt

KVL:
$\underline{V} = V_1 + V_{emf}$ ← Part ⓐ 1pt
$= -V_{emf}\left(\frac{1}{1+2}\right) + V_{emf} = \underline{\underline{66.7\ mV}}$ ← 1pt

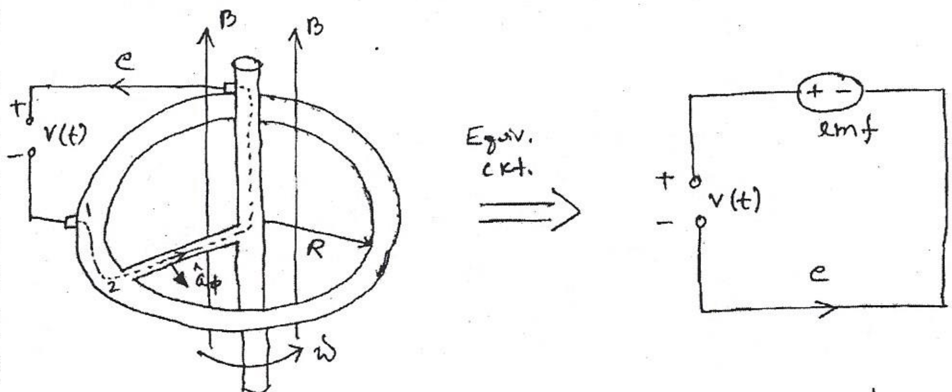
ⓒ



1pt

$\underline{V} = V_{emf}\left(\frac{2}{2+1}\right) = \underline{\underline{66.7\ mV}}$ ← 2pt.

[5pts] Prob. 3



Here, $\omega = \frac{\text{rad}}{\text{s}} = \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{\text{rev}}{\text{s}} \Rightarrow \omega = 2\pi N \left[\frac{\text{rad}}{\text{s}}\right]$

$\bar{v} = \omega r \hat{a}_\varphi = 2\pi N r \hat{a}_\varphi$; $d\bar{l} = -\hat{a}_r \, dr$ 1pt

Faraday's law and motional emf:

$$\text{emf} = \underbrace{\int_s \frac{d\bar{B}}{dt} \cdot d\bar{s}}_{=0} + \underbrace{\oint_c (\bar{v} \times \bar{B}) \, d\bar{l}}_{\text{motional emf}}$$

= 0 ; since B is not varying with time

$\therefore \text{emf} = \int_0^R (2\pi N r \cdot \hat{a}_\varphi \times B \hat{a}_z) \times (-\hat{a}_r \cdot dr)$; but $\hat{a}_\varphi \times \hat{a}_z = \hat{a}_r$ 1pt

$= \int_0^R -2\pi N r \cdot B \cdot dr = -2\pi N B \cdot \frac{R^2}{2}$ 1pt

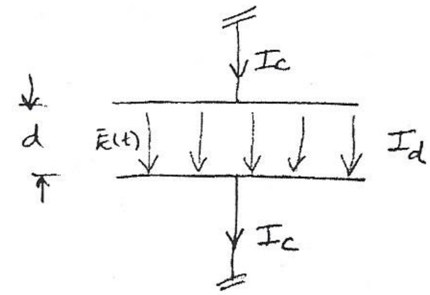
$= -\pi N B R^2$ 1pt

$\therefore V(t) = +\text{emf} = -\pi N B R^2$ V 1pt

See equivalent circuit above.

The only non-zero contribution. From $1 \to 2$, $\bar{v} \times \bar{B} \perp d\bar{l}$, so no contribution to integral, while for the remainder of close contour c is not rotating, thus $\bar{v} = 0$

[10 pts] Prob. 4



Here, $d = 0.04\ mm$

$= 0.04 \times 10^{-3}\ m$

$f = 15\ MHz$

$= 15 \times 10^{6}\ Hz$

$S = 10^{-3}\ m^2$

$V = 10\ V$

Now, displacement current, $\vec{J}_d = \frac{d\vec{D}}{dt}$ ← 1pt

In phasor form, $\vec{J}_d = j\omega \vec{D}$

$= j\omega \epsilon \vec{E}$

But, $|\vec{E}| = \frac{V}{d}$ ← 1pt

$= \frac{10}{0.04 \times 10^{-3}}$

$= 250 \times 10^{3}\ V/m$ ← 1pt

for Teflon, $\epsilon_r \approx 2.05$

So, $|\vec{J}_d| = \omega \epsilon |\vec{E}|$

$= 2\pi f\ \epsilon_r \epsilon_0\ 250 \times 10^{3}$ ← 1pt

$= 2\pi \times 15 \times 10^{6} \times 2.05 \times 8.854 \times 10^{-12} \times 250 \times 10^{3}$ ← 1pt

$= 427.67\ A/m^2$ ← 1pt

Now, $|I_d| = |\vec{J}_d| \times s$

$$= \underline{\underline{427.67 \times 10^{-3}}}\ A$$

2pts

The conduction current in the leads of the capacitor must equal to the displacement current in the capacitor.

$\therefore \underline{\underline{I_c}} = I_d = \underline{\underline{427.67}} \times 10^{-3}\ A$

2pts

[8 pts] Prob. 5

$$\text{Loss tangent} = \frac{\text{magnitude of conduction current}}{\text{magnitude of displacement current}} = \frac{|J_c|}{|J_d|} = \frac{|\sigma E|}{|j\omega\epsilon E|}$$

① Dry, sandy soil: which leads to: $\tan\delta = \dfrac{\sigma}{\omega\epsilon_r\epsilon_0}$

60 Hz:

$\sigma \approx 2.3\times10^{-9}$ S/m ; $\epsilon_r \approx 3.45$

$$\tan\delta = \frac{2.3\times10^{-9}}{2\pi\times60\times3.45\times\epsilon_0}$$

1pt

1pt $= \dfrac{2.3\times10^{-9}}{2\pi\times60\times3.45\times8.854\times10^{-12}}$

~~1pt~~

$= 0.199727$

10 GHz:

$\sigma \approx 5.6\times10^{-3}$ S/m ; $\epsilon_r \approx 2.5$

$$\tan\delta = \frac{5.6\times10^{-3}}{2\pi\times10\times10^{9}\times2.5\times8.854\times10^{-12}}$$ 1pt 1pt

$= 0.00402$

~~1pt~~

② Distilled water:

60 Hz

$\sigma \approx 1.1\times10^{-8}$ S/m ; $\epsilon_r \approx 81$

$$\therefore \tan\delta = \frac{1.1\times10^{-8}}{2\pi\times60\times81\times8.854\times10^{-12}}$$ 1pt

~~1pt~~ $= 0.0406852$

10 GHz

$\sigma \approx 5.6$ S/m ; $\epsilon_r \approx 50$ 1pt

$$\tan\delta = \frac{5.6}{2\pi\times10\times10^{9}\times50\times8.854\times10^{-12}}$$

$= 0.2013$ ~~1pt~~

③ copper:

60 Hz

$\sigma \approx 5.8\times10^{7}$ S/m ; $\epsilon_r \approx 1$

$$\therefore \tan\delta = \frac{5.8\times10^{7}}{2\pi\times60\times1\times8.854\times10^{-12}}$$

1pt $= 1.7376\times10^{16}$

10 GHz

$\sigma = 5.8\times10^{7}$ S/m ; $\epsilon_r \approx 1$

$$\therefore \tan\delta = \frac{5.8\times10^{7}}{2\pi\times10\times10^{9}\times1\times8.854\times10^{-12}}$$

$= 1.0425\times10^{8}$ 1pt