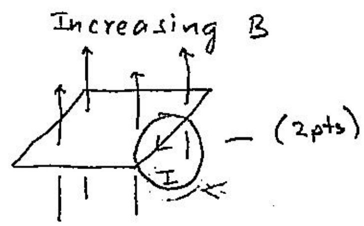
Prob. 1 - [8pts]

EE 382 - Applied EM

Homework #2

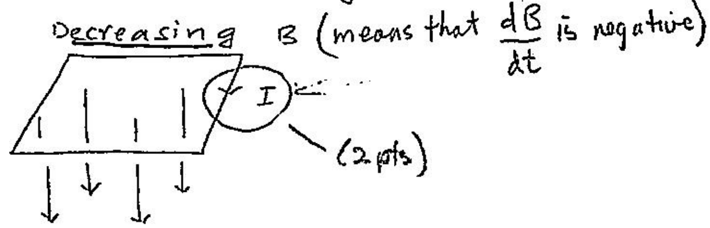
(a)



(b) No induced current (2pts) since the $\bar{B}$-field is not changing with time.

(c) No induced current (2pts) since the plane of $\vec{B}$-field is parallel with the loop, i.e., no flux-linkage.

(d)

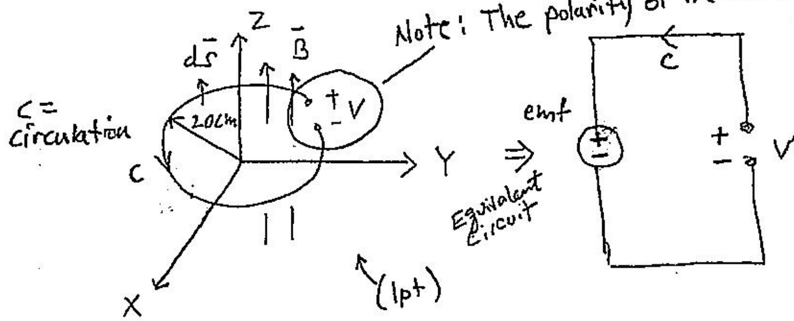


Prob. 2 EE 382 — Applied EM

[7pts] Homework #2

Text Problem 9.1

Note: The polarity of the answer will change if the polarity of the terminals is defined the opposite way.



$\vec{B} = 10\cos(377t)\,\hat{a}_z \ \text{mWb/m}^2$

By Faraday's law,

$emf = -\dfrac{d\Psi_m}{dt}$ (1pt)

$= -\dfrac{d}{dt}\int_s \vec{B}\cdot d\vec{s}$ where $d\vec{s} = \rho\, d\rho\, d\phi\, \hat{a}_z$ (1pt)

$= -\dfrac{d}{dt}\int_{\rho=0}^{0.2}\int_{\phi=0}^{2\pi} 10\cos(377t) \times \rho\, d\rho\, d\phi \times 10^{-3}$

$= -10\dfrac{d}{dt}\cos(377t)\int_{\rho=0}^{0.2}\int_{\phi=0}^{2\pi}(\rho\, d\rho\, d\phi) \times 10^{-3}$

$= -10 \times 377 \times (-\sin 377t)\ \underbrace{\left[\dfrac{\rho^2}{2}\right]_0^{0.2}\left[\phi\right]_0^{2\pi}}_{\text{area of circle} = \pi\rho^2} \times 10^{-3}$ (2pt)

$= 3770\sin 377t \times 0.04 \times \pi \times 10^{-3}$ (1pt)

$= 0.474\sin(377t)\ V$ — (1pt)

$V = \underline{\underline{0.474\sin(377t)\ V}}$

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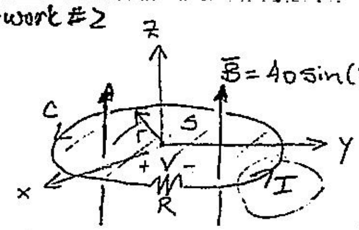
Homework #2

Text Prob. 9.3

$r = 0.2\ m$

$R = 4\,\Omega$

$\bar{B} = 40 \sin(10^4 t)\ \hat{a}_z\ \frac{mWb}{m^2}$



— (1pt)

By Faraday's Law: $emf = -\frac{d\Psi_m}{dt} = -\frac{d}{dt}\int_{S(c)} \bar{B}\cdot d\bar{S}$ —

Note: the polarity of the answer will change if I is defined in the other direction

with C as chosen above, $d\bar{S} = \hat{a}_z\, dx\, dy$ (1pt)

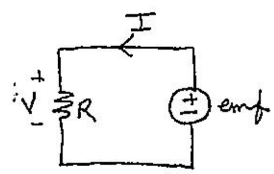
Since $\bar{B}$ is uniform, then $\int_S \bar{B}\cdot d\bar{S} = B\cdot A = B\cdot \pi r^2$ — (1pt)

$= \pi(0.2)^2 \cdot 40 \sin(10^4 t)\ mWb$

$\therefore\ emf = -\frac{d}{dt}\, 5.027 \sin(10^4 t)\ mWb = -5.027\times 10^4 \cos(10^4 t)\ mWb$

(1pt)

Equivalent ckt:



From this ckt we see that $V = emf$.

$\therefore\ I = \frac{V}{R} = \frac{emf}{R} = -\frac{50.27}{4} \cos(10^4 t)\ A$ — (1pt)

or $\underline{\underline{I = -12.57 \cos(10^4 t)\ A}}$ — (1pt)

(1pt)

with the positive direction as shown above.

Prob. 4

[8pts.]

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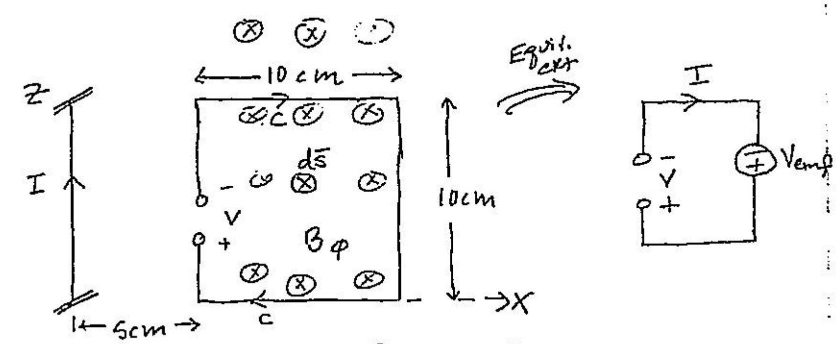
Homework #2



(1pt)

Given,

Amplitude = 3A

f = 4 KHz

$\therefore I = 3\cos(\omega t)$

$= 3\cos(2\pi f t)$ A

$\Rightarrow I(t) = 3\cos(2\pi \times 4 \times 10^3 t)$ — (1pt)

$= 3\cos(8\pi \times 10^3 t)$ A

$V_{emf} = -\frac{d\Psi_m}{dt}$ where $\Psi_m = \int_S \vec{B} \cdot d\vec{s}$ and

$d\bar{s} = d\rho\, dz\, \hat{a}_\phi$

From Biot-Savart law

(1pt) — $\bar{B} = \frac{\mu_0 I}{2\pi\rho}\hat{a}_\phi \ [\frac{Wb}{m^2}]$

— (1pt)

$\therefore \Psi_m = \int_0^{0.1}\int_{0.05}^{0.15} \frac{4\pi \times 10^{-7}}{2\pi\rho} 3\cos(8\pi \times 10^3 t)\, d\rho\, dz$

$$= \frac{4\times10^{-7}}{2} \int_0^{0.1} dz \int_{0.05}^{0.15} \frac{1}{\rho}\, d\rho \cdot 3\cos(8\pi\times10^3 t)$$

— (1pt)

$$= 2\times10^{-7}\times0.1\times\ln\left(\frac{0.15}{0.05}\right) 3\cos(8\pi\times10^3 t)$$

$$= 66\times10^{-9}\cos(8\pi\times10^3 t)\ \text{Wb}$$ — (1pt)

$$V_{emf} = -\frac{d\Psi_m}{dt}$$

— (1pt)

$$= -66\times10^{-9}\times(-)\sin(8\pi\times10^3 t)\times(8\pi\times10^3)$$

$$= 1.659\times10^{-3}\sin(8\pi\times10^3 t)\ \text{V}$$ — (1pt)

$= V \leftarrow$ from figure above. Hence, polarity of V is as shown in figure.

$|V| = 1.659$ mV

(1pt)

Problem 5

[10pts].

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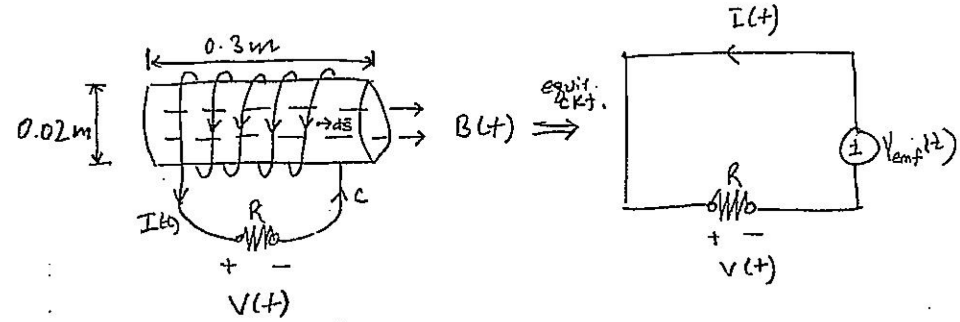
Homework #2



Given, N = 500 turns

$$B(t) = 0.03 \cos(2\pi \cdot 50{,}000\, t)\ \text{Wb/m}^2$$

$$\Psi_m = \int_S \vec{B}\, d\vec{s} = B \cdot A$$

$d\vec{s}$ in same direction as $\vec{B}$! Tricky :)

$$= 0.03 \cos(2\pi \cdot 50{,}000 t) \cdot \pi (0.01)^2$$

$$= 9.425 \times 10^{-6} \cos(2\pi \cdot 50{,}000 t)\ \text{Wb} \quad \text{— (1 pt)}$$

Flux Linkage

$$\lambda = N \Psi_m$$

$$= 500 \times 9.425 \times 10^{-6} \cos(2\pi \cdot 50{,}000 t)$$

$$= 4.712 \times 10^{-3} \cos(2\pi\, 50{,}000 t)\ \text{Wb} \quad \text{— (1 pt)}$$

$$V_{emf} = -\frac{d\lambda}{dt} = -N \frac{d\Psi_m}{dt} \quad \text{— (1 pt)}$$

$$= -\frac{d}{dt}\left[4.712 \times 10^{-3} \cos(2\pi \cdot 50{,}000 t)\right] \quad \text{— (1 pt)}$$

$$= -4.712 \times 10^{-3} (-)(\sin(2\pi \cdot 50{,}000 t) \cdot 2\pi \cdot 50{,}000$$

From equiv. ckt. above

$$= \underline{1.48 \times 10^{3} \sin(2\pi \cdot 50{,}000 t)\ V = V(t)} \quad \text{— (1 pt)}$$

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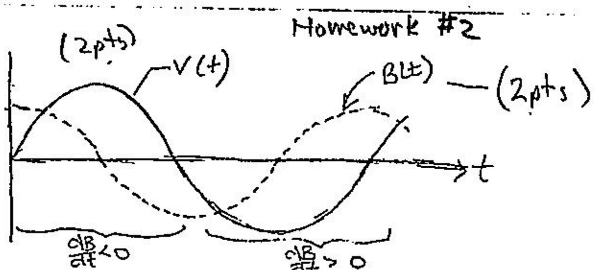

With a large resistor across V(t), the reactance of the circuit can be ignored and $I(t) \propto V(t)$

Lenz's law is satisfied from the above graph:

While $\frac{dB}{dt} < 0$, $V > 0$ and the induced magnetic field (using RHR) will oppose $\frac{d\Psi_m}{dt}$. Conversely, for $\frac{dB}{dt} > 0$, $V < 0$ and the induced magnetic flux will again oppose the change in Ψ_m.

(1pt)