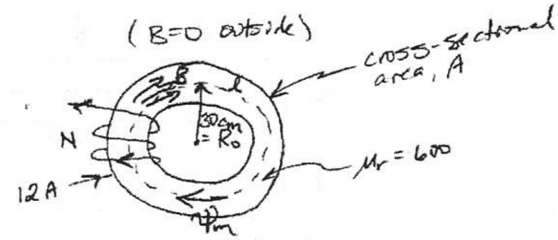
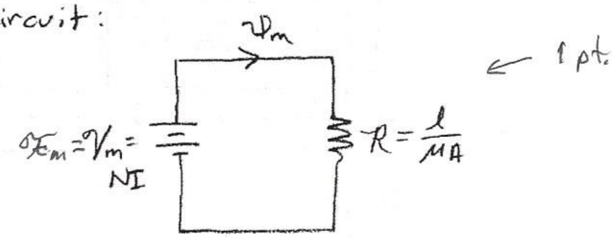
Text Prob. 8.49 [4pts]

Given a cobalt toroid:



Equivalent magnetic circuit:



From this circuit, we determine $V_m = \Psi_m \cdot \mathcal{R}$ (1)

where $\Psi_m = B \cdot A$ ← 1 pt

& $\mathcal{R} = \frac{l}{\mu A} = \frac{2\pi R_0}{\mu_0 \mu_r A}$ ← 1 pt

∴ From (1) $NI = BA \cdot \frac{2\pi R_0}{\mu_0 \mu_r A}$

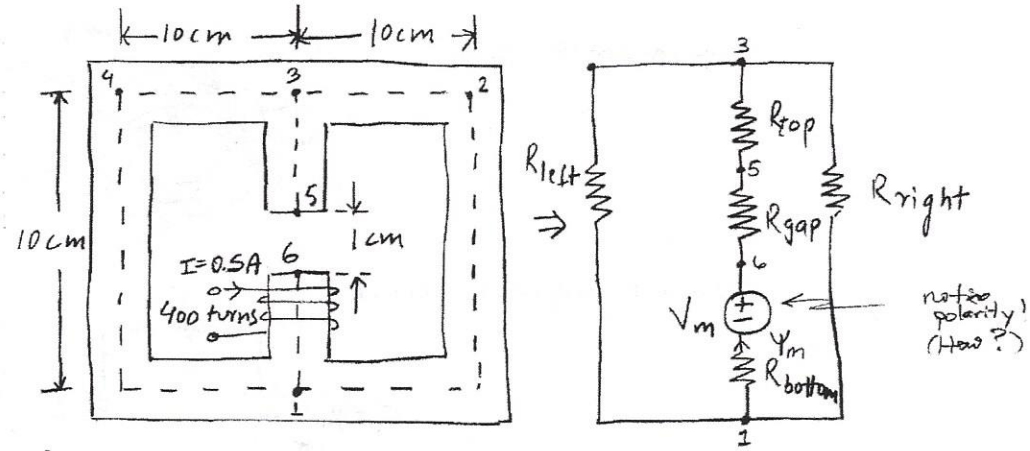
So $N = \frac{2\pi R_0 \cdot B}{\mu_0 \mu_r I} = \frac{2\pi \cdot 0.3 \cdot 1.5}{4\pi \times 10^{-7} \cdot 600 \cdot 12}$ ← 1 pt

$N = 312.5 \approx 313$ turns

4

note: $\mathcal{F}$ may be substituted for V in any equation.

Text Prob. 8.50



Given,

$I = \cancel{1}\ 0.5\ A$

$N = 400$ turns

$\mu = 500\mu_0$

$A = 10\ cm^2$

Now, $R = \dfrac{l}{\mu A}$

$$R_{left} = R_{right} = \frac{0.1 + 0.1 + 0.1}{500 \times 4\pi \times 10^{-7} \times 10 \times 10^{-4}}$$

$$= 4.775 \times 10^5\ H^{-1}$$

$$R_{top} = R_{bottom} = \frac{(10-1) \times 10^{-2}/2}{500 \times 4\pi \times 10^{-7} \times 10 \times 10^{-4}}$$

$$= 7.162 \times 10^4\ H^{-1}$$

1pt

$$R_{gap} = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}}$$ ($\because \mu = \mu_0$ for gap)

$= 7.958 \times 10^{6}\ H^{-1}$ ⟵ 1pt

$R_{total} = R_{top} + R_{bottom} + R_{gap} + (R_{left} \,||\, R_{right})$

$= 8.340 \times 10^{6}\ H^{-1}$

Now, $V_m = NI$

$= 400 \times 0.5$ ⟵ 1pt

$= 200$ A·t

$$\Psi_m = \frac{V_m}{R_{total}}$$

$$= \frac{200}{8.340 \times 10^{6}}$$

$= 2.399 \times 10^{-5}$ Wb ⟵ 1pt

Also, $\Psi_m = B \cdot A$ where $B = \mu H$. In the air gap, neglecting fringing, then

$$\Rightarrow H = \frac{\Psi_m}{\mu_0 A}$$

$$= \frac{2.399 \times 10^{-5}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}}$$ ⟵ 1pt

$= 1.908 \times 10^{4}$ A/m [5] pointing upward [1]

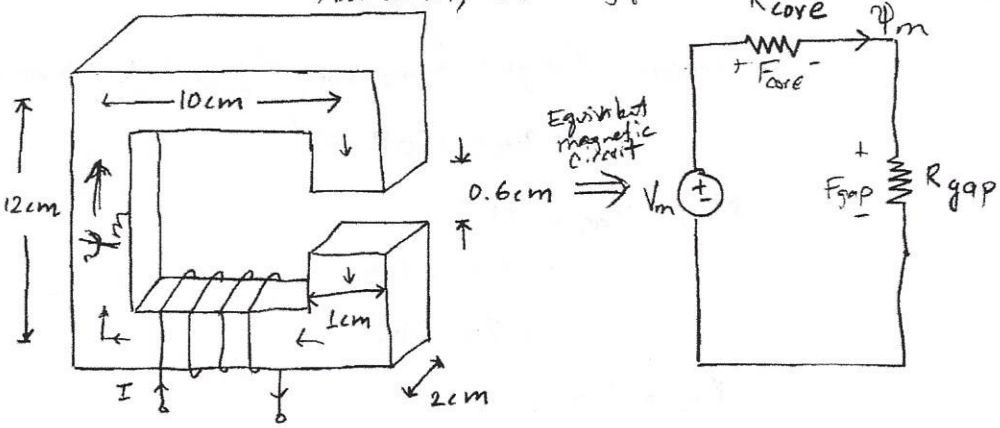
$mmf = \int_{gap} \vec{H} \cdot d\vec{l} = H \cdot l = 1.908 \times 10^{4} \times 0.01$

$= 190.8$ A·t $(= F_a)$ [2]

1pt for equation, 1pt for solution

neglecting fringing & flux leakage

Text prob. 8.51 – Also determine magnitude & direction of magnetic flux density in the air gap.



Given,

$I = 10A$

$N = 2{,}000$ turns

$A = 2\ cm^2$

$\mu_r = 1{,}500$

Now, $R_{core} = \dfrac{l}{\mu_0 \mu_r A} = \dfrac{0.1 + 0.1 + 0.12 + (0.12 - 0.006)}{4\pi \times 10^{-7} \times 1500 \times 2 \times 10^{-4}}$

$= 1.15 \times 10^6\ H^{-1}$

$R_{gap} = \dfrac{l_{gap}}{\mu_0 A} = \dfrac{0.006}{4\pi \times 10^{-7} \times 2 \times 10^{-4}} = 23.87 \times 10^6\ H^{-1}$

$R_{total} = R_{core} + R_{gap}$

$= 25.02 \times 10^6\ H^{-1}$

$V_m = NI$

$= 2{,}000 \times 10$

$= 20{,}000\ A \cdot t$

$$\Psi_m = \frac{V_m}{R_{total}} = \Psi_{core} = \Psi_{gap} = \frac{20{,}000}{25.02 \times 10^6} = 7.994 \times 10^{-4} \ \frac{Wb}{m^2}$$

2pts for first answer + 1pt for second answer

Using "mmf division", (analogous to voltage division)

$$F_{core} = \frac{R_{core}}{R_{gap} + R_{core}} \times V_m$$

$$= 919.3 \ A{\cdot}t$$

$$F_{gap} = \frac{R_{gap}}{R_{core} + R_{gap}} \times V_m$$

$$= 1.908 \times 10^4 \ A{\cdot}t$$

2pts for first correct answer + 1pt for second " "

$$B = \frac{\Psi_m}{A}$$

$$= \frac{7.994 \times 10^{-4}}{2 \times 10^{-4}}$$

$$= 3.997 \ Wb/m^2$$ ← Directed downward in the gap.

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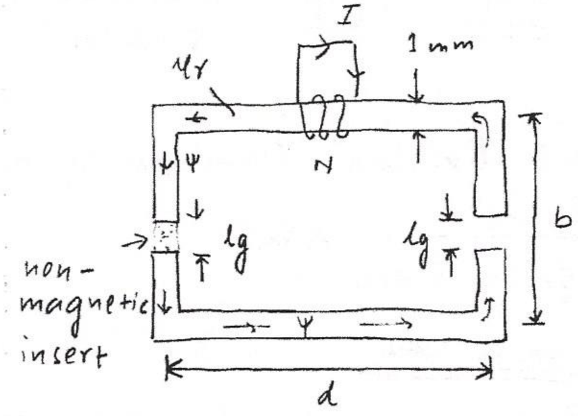


We know,

$B = \frac{\psi}{S}$ where $\psi = \frac{F}{R}$

Here, $S = 1\ mm^2$

$= 1 \times 10^{-6}\ m^2$

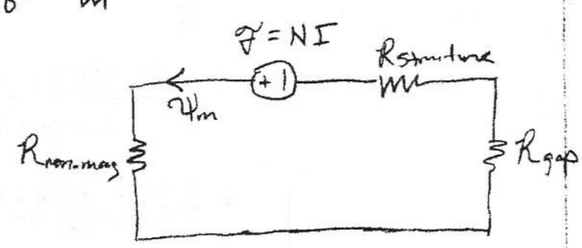
Again, $F = NI$

Here, $N = 300$ turns

$I = 5A$

$\therefore F = 300 \times 5$

$= 1500\ At$



1pt

Also, $R = R_{non\text{-}mag.\ insert} + 2 \cdot R_{structure} + R_{gap}$

And, $R_{non\text{-}mag.\ insert} = \frac{l_g}{\mu_0 \mu_r S}$

$= \frac{0.1 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 1 \times 10^{-6}}$

[$\because \mu_r$ for non-magnetic material is 1] ← 1pt

$= 79.57 \times 10^{6}\ H^{-1}$

Similarly, $R_{structure} = \frac{2d + 2(b - l_g)}{\mu_0 \mu_r S}$

$= \frac{2 \times 20 \times 10^{-2} + 2(6 \times 10^{-2} - 0.1 \times 10^{-3})}{4\pi \times 10^{-7} \times 750 \times 1 \times 10^{-6}}$

$= 551.52 \times 10^{6}\ H^{-1}$ ← 1pt

$R_{gap} = \frac{l_g}{\mu_0 S}$

$= 79.57 \times 10^{6}\ H^{-1}$ ← 1pt

$\Rightarrow R = 710.66 \times 10^{6}\ H^{-1}$

$\Rightarrow \Psi = \frac{1500}{710.66 \times 10^{6}}$

$= 2.11 \times 10^{-6}\ Wb$

And, $B = \frac{2.11 \times 10^{-6}}{1 \times 10^{-6}}$

$= 2.11\ \frac{Wb}{m^2}$ ↓ downward

} 1pt

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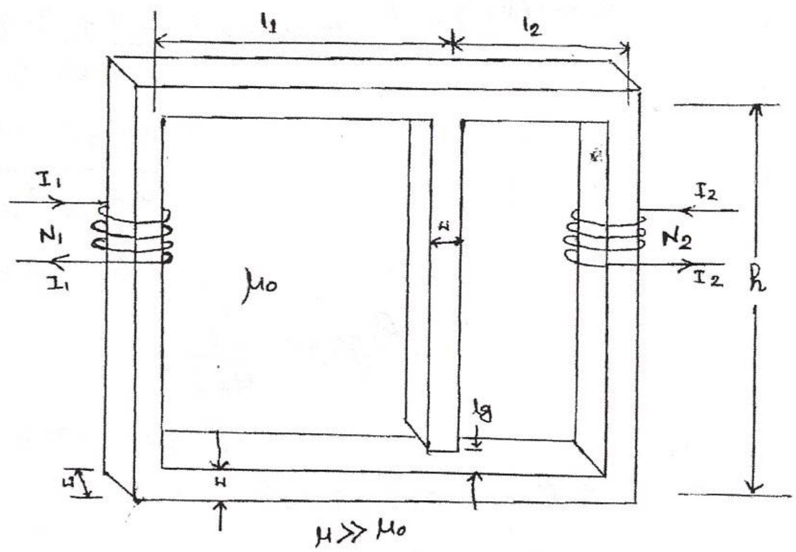
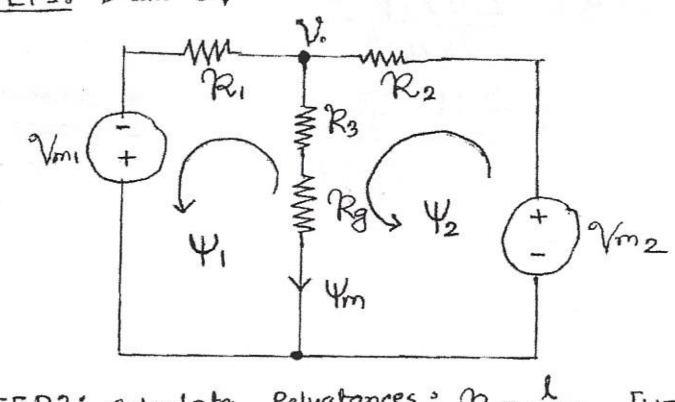


Given,

- $I_1 = 3A$
- $h = l_1 = 10$ cm
- $l_2 = 5$ cm
- $N_1 = 150$ turns
- $N_2 = 200$ turns
- $l_g = 1$ mm
- $w = 1$ cm
- $S = w^2 = 1 \times 10^{-4}$ m^2
- $\mu = 1{,}500\mu_0$

STEP1: Draw equivalent circuit



← 1pt

STEP2: Calculate Reluctances : $\mathcal{R} = \frac{l}{\mu S}$ $[H^{-1}]$

$$\mathcal{R}_1 = \frac{h + 2l_1}{\mu S} = \frac{0.1 + 2 \times 0.1}{1500 \times 4\pi \times 10^{-7} \times 1 \times 10^{-4}} = 1591.549 \times 10^3 \ [H^{-1}]$$

$$\mathcal{R}_2 = \frac{h + 2l_2}{\mu S} = \frac{0.1 + 2 \times 0.05}{1500 \times 4\pi \times 10^{-7} \times 1 \times 10^{-4}} = 1061.032 \times 10^3 \ [H^{-1}]$$

} 1pt.

$V_{m1} = N_1 I_1 = 150 \times 3 = 450$ At

If $B_{gap} = 0 \Rightarrow \Psi_m = \Psi_2 - \Psi_1 = 0 \quad \therefore V_0 = 0$ and

$\Psi_1 = \frac{V_{m1}}{R_1} = \frac{450}{1591.549 \times 10^3} = 282.743 \times 10^{-6}$ wb

$\Psi_1 - \Psi_2 = 0 , \quad \Psi_1 = \Psi_2$

} 1pt

$V_{m2} = N_2 I_2 = 200 \times I_2$ ← 1pt

Again with $V_0 = 0$ then ← 1pt

$V_{m2} = R_2 \Psi_2 = 1061.032 \times 10^3 \times 282.743 \times 10^{-6} = 300.283$ At

$\therefore \underline{\underline{I_2}} = \frac{300.283}{200} = \underline{\underline{1.501 \text{ A}}}$

5

Alternate solution:

In order for $\overline{B_{gap}} = 0$, then V_0 in the magnetic schematic must also be zero, because $\Psi_1 - \Psi_2 = 0$. ← 2pts

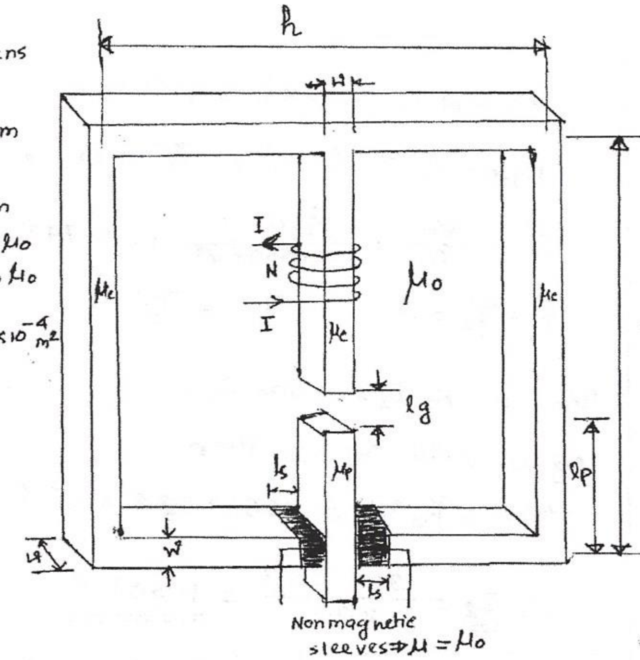
Therefore, $V_{m1} R_1 = V_{m2} R_2$

and $\frac{V_{m1}}{V_{m2}} = \frac{R_2}{R_1} = \frac{\frac{h+2l_1}{\mu S}}{\frac{h+2l_2}{\mu S}} = \frac{h+2l_1}{h+2l_2} = \frac{10+2\cdot 10}{10+2\cdot 5} = \frac{30}{20}$ ← 1pt

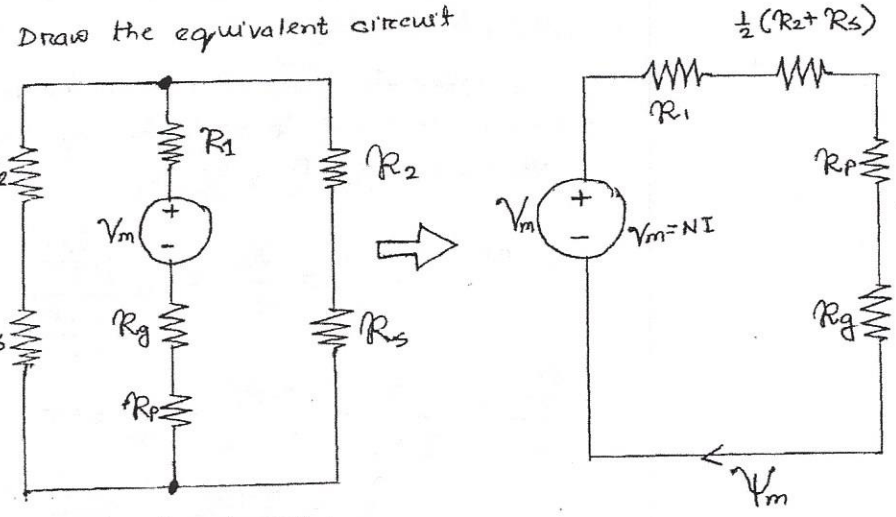
$\frac{V_{m1}}{V_{m2}} = \frac{I_1 N_1}{I_2 N_2} = \frac{3\cdot 150}{I_2 \cdot 200} = \frac{30}{20} = 1.5$ ← 1pt

$\underline{\underline{I_2}} = \frac{3\cdot 150}{1.5\cdot 200} = \frac{450}{300} = \underline{\underline{1.5A}}$ ← 1pt

- $I = 4A$
- $N = 150$ turns
- $h = 10\,cm$
- $l_p = w = 1\,cm$
- $l_g = 1\,mm$
- $l_s = 0.5\,mm$
- $\mu_c = 3{,}000\,\mu_0$
- $\mu_p = 2{,}000\,\mu_0$
- $S = w^2 = 1\times10^{-4}\,m^2$



STEP1: Draw the equivalent circuit



← 1pt

STEP2: calculate Reluctances;

$$R_1 = \frac{h - l_p - l_g}{3000\,\mu_0 S} = \frac{0.1 - 0.01 - 0.001}{3000\times4\pi\times10^{-7}\times1\times10^{-4}} = 236.079\times10^3\,[H^{-1}]$$

$$R_2 = \frac{\frac{h}{2} + h + \frac{h}{2} - (\frac{w}{2} + l_s)}{3000\,\mu_0 S} = \frac{2h - \frac{w}{2} - l_s}{3000\,\mu_0 S} = \frac{2\times0.1 - \frac{0.01}{2} - 0.5\times10^{-3}}{3000\times4\pi\times10^{-7}\times1\times10^{-4}}$$

$$= 515.927\times10^3\,[H^{-1}]$$

} 1pt

$\cdot \mathcal{R}_{os} = \dfrac{l_s}{\mu_o S} = \dfrac{0.5\times10^{-3}}{4\pi\times10^{-7}\times10^{-4}} = 3978.873\times10^3 [H^{-1}]$ ← 1pt

$\cdot \mathcal{R}_P = \dfrac{l_P}{2000\,\mu_o S} = \dfrac{1\times10^{-2}}{2000\times4\pi\times10^{-7}\times1\times10^{-4}} = 39.788\times10^3 [H^{-1}]$

$\cdot \mathcal{R}_g = \dfrac{l_g}{\mu_o S} = \dfrac{1\times10^{-3}}{4\pi\times10^{-7}\times1\times10^{-4}} = 7957.747\times10^3 [H^{-1}]$

} 1pt

Refering to the simplified equivalent circuit : ← 1pt

$\Psi_m = \dfrac{NI = \mathcal{V}_m}{\mathcal{R}_1 + \frac{1}{2}(\mathcal{R}_2 + \mathcal{R}_s) + \mathcal{R}_P + \mathcal{R}_g} = 57.246\times10^{-6} [Wb]$ 5

$\cdot |\bar{B}| = \dfrac{\Psi_m}{S} = \dfrac{57.246\times10^{-6}}{1\times10^{-4}} = 572.463\times10^{-3} \left[\dfrac{Wb}{m^2}\right]$ 2

1pt ↑ 1pt ↑

· Directed upward through airgap 1