## Maxwell's Equations

## Static Fields:

|  | Integral Form | Differential Form |
| :--- | :--- | :--- |
| Faraday's Law | $\oint_{c} \bar{E} \cdot d \bar{l}=0$ | $\bar{\nabla} \times \bar{E}=0$ |
| Ampere's Law | $\oint_{c} \bar{H} \cdot d \bar{l}=\int_{s} \bar{J} \cdot d \bar{s}$ | $\bar{\nabla} \times \bar{H}=\bar{J}$ |
| Gauss' Law | $\oint_{s} \bar{D} \cdot d \bar{s}=\int_{V} \rho_{v} d V$ | $\bar{\nabla} \cdot \bar{D}=\rho_{v}$ |
|  | $\oint_{s} \bar{B} \cdot d \bar{s}=0$ | $\bar{\nabla} \cdot \bar{B}=0$ |

## Time-Varying Fields:

Integral Form
Faraday's Law

$$
\oint_{c} \overline{\mathcal{E}} \cdot d \bar{l}=-\frac{d}{d t} \int_{s} \overline{\mathcal{B}} \cdot d \bar{s}
$$

Differential Form
$\bar{\nabla} \times \overline{\mathcal{E}}=-\frac{\partial \overline{\mathcal{B}}}{\partial t}$
Ampere's Law
Gauss' Law

$$
\begin{array}{ll}
\oint_{s} \overline{\mathcal{D}} \cdot d \bar{s}=\int_{V} \rho_{v} d V & \bar{\nabla} \cdot \overline{\mathcal{D}}=\rho_{1} \\
\oint_{s} \overline{\mathcal{B}} \cdot d \bar{s}=0 & \bar{\nabla} \cdot \overline{\mathcal{B}}=0
\end{array}
$$

## Time-Varying Fields, simple media, \& stationary circuits:

Integral Form
Faraday's Law $\quad \oint_{c} \overline{\mathcal{E}} \cdot d \bar{l}=-\mu \frac{d}{d t} \int_{\mathrm{s}} \overline{\mathcal{H}} \cdot d \bar{s}$
Ampere's Law $\quad \oint_{c} \overline{\mathcal{H}} \cdot d \bar{l}=\sigma \int_{s} \overline{\mathcal{E}} \cdot d \bar{s}+\varepsilon \frac{d}{d t} \int_{\mathrm{s}} \overline{\mathcal{E}} \cdot d \bar{s}$ $+\int_{s} \overline{\mathcal{J}} \cdot d \bar{s}$
Gauss' Law

$$
\begin{aligned}
& \oint_{s} \overline{\mathcal{E}} \cdot d \bar{s}=\frac{1}{\varepsilon} \int_{V} \rho_{v} d V \\
& \oint_{s} \overline{\mathcal{H}} \cdot d \bar{s}=0
\end{aligned}
$$

Differential Form
$\bar{\nabla} \times \overline{\mathcal{E}}=-\mu \frac{\partial \overline{\mathcal{H}}}{\partial t}$
$\bar{\nabla} \times \overline{\mathcal{H}}=\sigma \overline{\mathcal{E}}+\varepsilon \frac{\partial \overline{\mathcal{E}}}{\partial t}+\overline{\mathcal{J}}$
$\bar{\nabla} \cdot \overline{\mathcal{E}}=\frac{\rho_{v}}{\varepsilon}$
$\bar{\nabla} \cdot \overline{\mathcal{H}}=0$

## Sinusoidal Steady-State Time-Varying Fields:

Integral Form
Faraday's Law

$$
\oint_{c} \hat{E} \cdot d \bar{l}=-j \omega \int_{s}^{\hat{B}} \cdot d \bar{S}
$$

Ampere's Law $\quad \oint_{c} \hat{\bar{H}} \cdot d \bar{l}=\int_{s} \hat{\bar{J}} \cdot d \bar{s}+j \omega \int_{s} \hat{\bar{D}} \cdot d \bar{s} \quad \bar{\nabla} \times \hat{\bar{H}}=\hat{\bar{J}}+j \omega \hat{\bar{D}}$
Gauss' Law

$$
\begin{array}{ll}
\oint_{s} \hat{\bar{D}} \cdot d \bar{s}=\int_{V} \hat{\rho}_{v} d V & \bar{\nabla} \cdot \hat{\bar{D}}=\hat{\rho}_{v} \\
\oint_{s} \hat{\bar{B}} \cdot d \bar{s}=0 & \bar{\nabla} \cdot \hat{\bar{B}}=0
\end{array}
$$

$\bar{\nabla} \times \hat{\bar{E}}=-j \omega \hat{\bar{B}}$

## Sinusoidal Steady-State Time-Varying Fields \& Simple Media:

Integral Form
Faraday's Law $\quad \oint_{c} \hat{\bar{E}} \cdot d \bar{l}=-j \omega \mu \int_{\mathrm{s}} \hat{\bar{H}} \cdot d \bar{s}$
Ampere's Law $\quad \oint_{c} \hat{\bar{H}} \cdot d \bar{l}=(\sigma+j \omega \varepsilon) \int_{s} \hat{\bar{E}} \cdot d \bar{s}+\int_{s} \hat{\bar{J}} \cdot d \bar{s} \quad \bar{\nabla} \times \hat{\bar{H}}=(\sigma+j \omega \varepsilon) \hat{\bar{E}}+\hat{\bar{J}}$
Gauss' Law

$$
\oint_{s} \hat{\bar{E}} \cdot d \bar{s}=\frac{1}{\varepsilon} \int_{V} \hat{\rho}_{v} d V
$$

$$
\oint_{s} \hat{\bar{H}} \cdot d \bar{s}=0
$$

Differential Form

$$
\bar{\nabla} \times \hat{\bar{E}}=-j \omega \mu \hat{\bar{H}}
$$

$$
\bar{\nabla} \cdot \hat{\bar{E}}=\frac{\hat{\rho}_{v}}{\varepsilon}
$$

$$
\bar{\nabla} \cdot \hat{\bar{H}}=0
$$

## Other important relationships:

## Integral Form

$$
\underline{\text { Differential Form }}
$$

Eqn of Continuity /
Conservation of Charge

$$
\oint_{s} \overline{\mathcal{J}} \cdot d \bar{s}=-\frac{d}{d t} \int_{V} \rho_{v} d V
$$

$$
\bar{\nabla} \cdot \overline{\mathcal{J}}=-\frac{\partial \rho_{v}}{\partial t}
$$

Lorentz Force Eqn. $\quad \overline{\mathcal{F}}=q(\overline{\mathcal{E}}+\bar{u} \times \overline{\mathcal{B}})$
Constitutive Relations

$$
\overline{\mathcal{D}}=\varepsilon \overline{\mathcal{E}}=\varepsilon_{0} \overline{\mathcal{E}}+\overline{\mathcal{P}} \quad \overline{\mathcal{B}}=\mu \overline{\mathcal{H}}=\mu_{0}(\overline{\mathcal{H}}+\overline{\mathcal{M}})
$$

Ohm's Law

$$
\overline{\mathcal{J}}_{c}=\sigma \overline{\mathcal{E}}
$$

Electric
Boundary Conditions $\left\{\begin{array}{ccc}\text { Tangential- } & \overline{\mathcal{E}}_{1 t}=\overline{\mathcal{E}}_{2 t} \text { or } \hat{a}_{\mathrm{n} 12} \times\left(\overline{\mathcal{E}}_{2}-\overline{\mathcal{E}}_{1}\right)=0 & \begin{array}{c}\hat{a}_{\mathrm{n} 12} \times\left(\overline{\mathcal{H}}_{2}-\overline{\mathcal{H}}_{1}\right)=\overline{\mathcal{J}}_{s} \\ \text { Normal- }\end{array} \\ \hat{a}_{\mathrm{n} 12} \cdot\left(\overline{\mathcal{D}}_{2}-\overline{\mathcal{D}}_{1}\right)=\rho_{s} & \overline{\mathcal{B}}_{1 n}=\overline{\mathcal{B}}_{2 n} \text { or } \hat{a}_{\mathrm{n} 12} \cdot\left(\overline{\mathcal{B}}_{2}-\overline{\mathcal{B}}_{1}\right)=0\end{array}\right.$
where surface normal $\hat{a}_{\mathrm{n} 12}$ points from region 1 into region 2 , and $\mathcal{B}_{1 n} \& \mathcal{D}_{1 n}$ point away from boundry while $\mathcal{B}_{2 n} \& \mathcal{D}_{2 n}$ point toward from boundry

$$
\text { Poynting Vector } \quad \overline{\mathcal{S}}=\overline{\mathcal{E}} \times \overline{\mathcal{H}}
$$

## Poynting Theorem

Differential Form- $\quad-\bar{\nabla} \cdot \overline{\mathcal{S}}=\overline{\mathcal{E}} \cdot \overline{\mathbb{L}}+\overline{\mathcal{E}} \cdot \frac{\partial \overline{\mathcal{D}}}{\partial t}+\overline{\mathcal{H}} \cdot \frac{\partial \overline{\mathcal{B}}}{\partial t}$
Integral Form-

$$
-\oint_{s} \overline{\mathcal{S}} \cdot d \bar{s}=\int_{V} \overline{\mathcal{E}} \cdot \overline{\mathbb{}}{ }^{2} V+\int_{V}\left(\overline{\mathcal{E}} \cdot \frac{\partial \overline{\mathcal{D}}}{\partial t}\right) d V+\int_{V}\left(\overline{\mathcal{H}} \cdot \frac{\partial \overline{\mathcal{B}}}{\partial t}\right) d V
$$

