

### Rectangular/Cartesian Coordinates ( $x, y, z$ )

$$d\vec{l} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$$

$$d\vec{s}_x = dy dz \hat{a}_x, \quad d\vec{s}_y = dx dz \hat{a}_y, \quad d\vec{s}_z = dx dy \hat{a}_z$$

$$dV = dx dy dz, \quad \vec{r} = x \hat{a}_x + y \hat{a}_y + z \hat{a}_z$$

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\begin{aligned} \vec{\nabla} \times \vec{A} = & \hat{a}_x \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{a}_y \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \\ & + \hat{a}_z \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \end{aligned}$$

### Spherical Coordinates ( $r, \theta, \phi$ )

$$d\vec{l} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin \theta d\phi \hat{a}_\phi$$

$$d\vec{s}_r = r^2 \sin \theta d\theta d\phi \hat{a}_r, \quad d\vec{s}_\theta = r \sin \theta dr d\phi \hat{a}_\theta, \quad d\vec{s}_\phi = r dr d\theta \hat{a}_\phi$$

$$dV = r^2 \sin \theta dr d\theta d\phi, \quad \vec{r} = r \hat{a}_r$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{r^2} \frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (\sin \theta A_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\begin{aligned} \vec{\nabla} \times \vec{A} = & \hat{a}_r \left\{ \frac{1}{r \sin \theta} \left[ \frac{\partial (\sin \theta A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right] \right\} + \hat{a}_\theta \left[ \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial (r A_\phi)}{\partial r} \right] \\ & + \hat{a}_\phi \left\{ \frac{1}{r} \left[ \frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \right\} \end{aligned}$$

### Rectangular/Cartesian Coordinates ( $x, y, z$ ) $\Leftrightarrow$ Cylindrical Coordinates ( $r, \phi, z$ )

$$\hat{a}_x \cdot \hat{a}_r = \cos \phi \quad \hat{a}_y \cdot \hat{a}_\phi = \cos \phi, \quad \hat{a}_x \cdot \hat{a}_\phi = -\sin \phi \quad \hat{a}_z \cdot \hat{a}_r = 0 \quad \hat{a}_y \cdot \hat{a}_r = \sin \phi \quad \hat{a}_z \cdot \hat{a}_\phi = 0$$

### Rectangular/Cartesian Coordinates ( $x, y, z$ ) $\Leftrightarrow$ Spherical Coordinates ( $r, \theta, \phi$ )

$$\begin{aligned} \hat{a}_r \cdot \hat{a}_x &= \sin \theta \cos \phi, \quad \hat{a}_r \cdot \hat{a}_y = \sin \theta \sin \phi, \quad \hat{a}_r \cdot \hat{a}_z = \cos \theta, \quad \hat{a}_\theta \cdot \hat{a}_x = \cos \theta \cos \phi, \quad \hat{a}_\theta \cdot \hat{a}_y = \cos \theta \sin \phi, \quad \hat{a}_\theta \cdot \hat{a}_z = -\sin \theta \\ \hat{a}_\phi \cdot \hat{a}_x &= -\sin \phi, \quad \hat{a}_\phi \cdot \hat{a}_y = \cos \phi, \quad \hat{a}_\phi \cdot \hat{a}_z = 0 \end{aligned}$$

Eqn of Continuity /  
Conservation of Charge

<u>Integral Form</u>	<u>Differential Form</u>
$\oint_s \bar{\mathcal{J}} \cdot d\bar{s} = -\frac{d}{dt} \int_V \rho_v dV$	$\bar{\nabla} \cdot \bar{\mathcal{J}} = -\frac{\partial \rho_v}{\partial t}$

Lorentz Force Eqn.

$$\bar{\mathcal{F}} = q(\bar{\mathcal{E}} + \bar{u} \times \bar{\mathcal{B}})$$

Constitutive Relations

$$\bar{\mathcal{D}} = \epsilon \bar{\mathcal{E}} = \epsilon_0 \bar{\mathcal{E}} + \bar{\mathcal{P}} \quad \bar{\mathcal{B}} = \mu \bar{\mathcal{H}} = \mu_0 (\bar{\mathcal{H}} + \bar{\mathcal{M}})$$

Ohm's Law

$$\bar{\mathcal{J}}_c = \sigma \bar{\mathcal{E}}$$

Boundary Conditions

Tangential- $\bar{\mathcal{E}}_{1t} = \bar{\mathcal{E}}_{2t}$ or $\hat{a}_{n12} \times (\bar{\mathcal{E}}_2 - \bar{\mathcal{E}}_1) = 0$	<u>Electric</u>	<u>Magnetic</u>
Normal- $\hat{a}_{n12} \cdot (\bar{\mathcal{D}}_2 - \bar{\mathcal{D}}_1) = \rho_s$	$\hat{a}_{n12} \times (\bar{\mathcal{H}}_2 - \bar{\mathcal{H}}_1) = \bar{\mathcal{J}}_s$	$\bar{\mathcal{B}}_{1n} = \bar{\mathcal{B}}_{2n}$ or $\hat{a}_{n12} \cdot (\bar{\mathcal{B}}_2 - \bar{\mathcal{B}}_1) = 0$

where surface normal  $\hat{a}_{n12}$  points from region 1 into region 2, and  $\mathcal{B}_{1n}$  &  $\mathcal{D}_{1n}$  point away from boundary

while  $\mathcal{B}_{2n}$  &  $\mathcal{D}_{2n}$  point toward from boundary

### Cylindrical Coordinates ( $r, \phi, z$ )

$$d\vec{l} = dr \hat{a}_r + r d\phi \hat{a}_\phi + dz \hat{a}_z$$

$$d\vec{s}_r = rd\phi dz \hat{a}_r, \quad d\vec{s}_\phi = r dz \hat{a}_\phi, \quad d\vec{s}_z = r dr d\phi \hat{a}_z$$

$$\begin{aligned} \vec{\nabla} \cdot \vec{A} = & \frac{1}{r} \frac{\partial (r A_r)}{\partial r} + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \end{aligned}$$

$$\begin{aligned} \vec{\nabla} \times \vec{A} = & \hat{a}_r \left( \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{a}_\phi \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \\ & + \hat{a}_z \left( \frac{1}{r} \frac{\partial (r A_\phi)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \phi} \right) \end{aligned}$$

### Trigonometric Identities

$$\sin(-A) = -\sin A, \quad \cos(-A) = \cos A$$

$$\cos(A - 90^\circ) = \sin A$$

$$\cos(A + 90^\circ) = -\sin A$$

### Time-Varying Fields:

	Integral Form	Differential Form	Integral Form	Differential Form	
Faraday's Law	$\oint_c \bar{\mathcal{E}} \cdot d\bar{l} = -\frac{d}{dt} \int_s \bar{\mathcal{B}} \cdot d\bar{s}$	$\bar{\nabla} \times \bar{\mathcal{E}} = -\frac{\partial \bar{\mathcal{B}}}{\partial t}$	Faraday's Law	$\oint_c \bar{\mathcal{E}} \cdot d\bar{l} = -\mu \frac{d}{dt} \int_s \bar{\mathcal{H}} \cdot d\bar{s}$	$\bar{\nabla} \times \bar{\mathcal{E}} = -\mu \frac{\partial \bar{\mathcal{H}}}{\partial t}$
Ampere's Law	$\oint_c \bar{\mathcal{H}} \cdot d\bar{l} = \int_s \bar{\mathcal{J}} \cdot d\bar{s} + \int_s \frac{\partial \bar{\mathcal{D}}}{\partial t} \cdot d\bar{s}$	$\bar{\nabla} \times \bar{\mathcal{H}} = \bar{\mathcal{J}} + \frac{\partial \bar{\mathcal{D}}}{\partial t}$	Ampere's Law	$\oint_c \bar{\mathcal{H}} \cdot d\bar{l} = \sigma \int_s \bar{\mathcal{E}} \cdot d\bar{s} + \varepsilon \frac{d}{dt} \int_s \bar{\mathcal{E}} \cdot d\bar{s} + \int_s \bar{\mathcal{J}} \cdot d\bar{s}$	$\bar{\nabla} \times \bar{\mathcal{H}} = \sigma \bar{\mathcal{E}} + \varepsilon \frac{\partial \bar{\mathcal{E}}}{\partial t} + \bar{\mathcal{J}}$
Gauss' Law	$\oint_s \bar{\mathcal{D}} \cdot d\bar{s} = \int_v \rho_v dV$	$\bar{\nabla} \cdot \bar{\mathcal{D}} = \rho_v$	Gauss' Law	$\oint_s \bar{\mathcal{E}} \cdot d\bar{s} = \frac{1}{\varepsilon} \int_v \rho_v dV$	$\bar{\nabla} \cdot \bar{\mathcal{E}} = \frac{\rho_v}{\varepsilon}$
	$\oint_s \bar{\mathcal{B}} \cdot d\bar{s} = 0$	$\bar{\nabla} \cdot \bar{\mathcal{B}} = 0$		$\oint_s \bar{\mathcal{H}} \cdot d\bar{s} = 0$	$\bar{\nabla} \cdot \bar{\mathcal{H}} = 0$

### Sinusoidal Steady-State Time-Varying Fields:

	Integral Form	Differential Form
Faraday's Law	$\oint_c \hat{\bar{E}} \cdot d\bar{l} = -j\omega \int_s \hat{\bar{B}} \cdot d\bar{s}$	$\bar{\nabla} \times \hat{\bar{E}} = -j\omega \hat{\bar{B}}$
Ampere's Law	$\oint_c \hat{\bar{H}} \cdot d\bar{l} = \int_s \hat{\bar{J}} \cdot d\bar{s} + j\omega \int_s \hat{\bar{D}} \cdot d\bar{s}$	$\bar{\nabla} \times \hat{\bar{H}} = \hat{\bar{J}} + j\omega \hat{\bar{D}}$
Gauss' Law	$\oint_s \hat{\bar{D}} \cdot d\bar{s} = \int_v \hat{\rho}_v dV$	$\bar{\nabla} \cdot \hat{\bar{D}} = \hat{\rho}_v$
	$\oint_s \hat{\bar{B}} \cdot d\bar{s} = 0$	$\bar{\nabla} \cdot \hat{\bar{B}} = 0$

### Time-Varying Fields, simple media, & stationary circuits:

	Integral Form	Differential Form
Faraday's Law	$\oint_c \bar{\mathcal{E}} \cdot d\bar{l} = -\mu \frac{d}{dt} \int_s \bar{\mathcal{H}} \cdot d\bar{s}$	$\bar{\nabla} \times \bar{\mathcal{E}} = -\mu \frac{\partial \bar{\mathcal{H}}}{\partial t}$
Ampere's Law	$\oint_c \bar{\mathcal{H}} \cdot d\bar{l} = \sigma \int_s \bar{\mathcal{E}} \cdot d\bar{s} + \varepsilon \frac{d}{dt} \int_s \bar{\mathcal{E}} \cdot d\bar{s} + \int_s \bar{\mathcal{J}} \cdot d\bar{s}$	$\bar{\nabla} \times \bar{\mathcal{H}} = \sigma \bar{\mathcal{E}} + \varepsilon \frac{\partial \bar{\mathcal{E}}}{\partial t} + \bar{\mathcal{J}}$
Gauss' Law	$\oint_s \bar{\mathcal{E}} \cdot d\bar{s} = \frac{1}{\varepsilon} \int_v \rho_v dV$	$\bar{\nabla} \cdot \bar{\mathcal{E}} = \frac{\rho_v}{\varepsilon}$
	$\oint_s \bar{\mathcal{H}} \cdot d\bar{s} = 0$	$\bar{\nabla} \cdot \bar{\mathcal{H}} = 0$

### Sinusoidal Steady-State Time-Varying Fields & Simple Media:

	Integral Form	Differential Form
Faraday's Law	$\oint_c \hat{\bar{E}} \cdot d\bar{l} = -j\omega \mu \int_s \hat{\bar{H}} \cdot d\bar{s}$	$\bar{\nabla} \times \hat{\bar{E}} = -j\omega \mu \hat{\bar{H}}$
Ampere's Law	$\oint_c \hat{\bar{H}} \cdot d\bar{l} = (\sigma + j\omega\varepsilon) \int_s \hat{\bar{E}} \cdot d\bar{s} + \int_s \hat{\bar{J}} \cdot d\bar{s}$	$\bar{\nabla} \times \hat{\bar{H}} = (\sigma + j\omega\varepsilon) \hat{\bar{E}} + \hat{\bar{J}}$
Gauss' Law	$\oint_s \hat{\bar{D}} \cdot d\bar{s} = \frac{1}{\varepsilon} \int_v \hat{\rho}_v dV$	$\bar{\nabla} \cdot \hat{\bar{E}} = \frac{\hat{\rho}_v}{\varepsilon}$
	$\oint_s \hat{\bar{H}} \cdot d\bar{s} = 0$	$\bar{\nabla} \cdot \hat{\bar{H}} = 0$