Rectangular/Cartesian Coordinates (x, y, z)

$$d\overline{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$$d\overline{s}_x = dy\,dz\,\hat{a}_x\,, \quad d\overline{s}_y = dx\,dz\,\hat{a}_y\,, \quad d\overline{s}_z = dx\,dy\,\hat{a}_z$$

$$dV = dx\,dy\,dz\,, \qquad \overline{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$$

$$\overline{\nabla}\Phi = \hat{a}_x\frac{\partial\Phi}{\partial x} + \hat{a}_y\frac{\partial\Phi}{\partial y} + \hat{a}_z\frac{\partial\Phi}{\partial z}\,, \quad \overline{\nabla}\bullet\overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\overline{\nabla}\times\overline{A} = \hat{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\overline{\nabla}^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}$$

Cylindrical Coordinates (ρ, ϕ, z)

$$d\overline{l} = d\rho\,\hat{a}_\rho + \rho d\phi\hat{a}_\phi + dz\hat{a}_z$$

$$d\overline{s}_\rho = \rho d\phi dz\,\hat{a}_\rho\,, \quad d\overline{s}_\phi = d\rho dz\hat{a}_\phi\,, \quad d\overline{s}_z = \rho d\rho d\phi\,\hat{a}_z$$

$$dV = \rho d\rho d\phi dz\,, \qquad \overline{r} = \rho\,\hat{a}_\rho + z\,\hat{a}_z$$

$$\overline{\nabla}\Phi = \hat{a}_\rho\frac{\partial\Phi}{\partial\rho} + \hat{a}_\phi\frac{1}{\rho}\frac{\partial\Phi}{\partial\phi} + \hat{a}_z\frac{\partial\Phi}{\partial z}\,, \quad \overline{\nabla}\bullet\overline{A} = \frac{1}{\rho}\frac{\partial\left(\rho A_\rho\right)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\overline{\nabla}\times\overline{A} = \hat{a}_\rho\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{a}_\phi\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right) + \hat{a}_z\left(\frac{1}{\rho}\frac{\partial\left(\rho A_\phi\right)}{\partial\rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial\phi}\right)$$

$$\overline{\nabla}^2\Phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$$

Spherical Coordinates (r, θ, ϕ)

$$d\overline{l} = dr\hat{a}_r + rd\theta\hat{a}_\theta + r\sin\theta\,d\phi\hat{a}_\phi$$

$$d\overline{s}_r = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r\,, \quad d\overline{s}_\theta = r\sin\theta\,dr\,d\phi\,\hat{a}_\theta\,, \quad d\overline{s}_\phi = r\,dr\,d\theta\,\hat{a}_\phi$$

$$dV = r^2\sin\theta dr d\theta d\phi\,, \qquad \overline{r} = r\,\hat{a}_r$$

$$\overline{\nabla}\Phi = \hat{a}_r\frac{\partial\Phi}{\partial r} + \hat{a}_\theta\frac{1}{r}\frac{\partial\Phi}{\partial\theta} + \hat{a}_\phi\frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}$$

$$\overline{\nabla}\bullet\overline{A} = \frac{1}{r^2}\frac{\partial\left(r^2A_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\left(\sin\theta A_\theta\right)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\overline{\nabla}\times\overline{A} = \hat{a}_r\left\{\frac{1}{r\sin\theta}\left[\frac{\partial\left(\sin\theta A_\phi\right)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\right\} + \hat{a}_\theta\left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial\left(rA_\phi\right)}{\partial r}\right] + \hat{a}_\phi\left\{\frac{1}{r}\left[\frac{\partial\left(rA_\theta\right)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\right\}$$

$$\overline{\nabla}^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$

Trigonometric Identities

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\cos A\cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\sin A\sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\sin A\cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\sin(-A) = -\sin A\,, \quad \cos(-A) = \cos A$$

$$\cos(A - 90^\circ) = \sin A$$

$$\cos(A + 90^\circ) = -\sin A$$

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\sin^2 A = \tfrac{1}{2} - \tfrac{1}{2}\cos 2A$$

$$\cos^2 A = \tfrac{1}{2} + \tfrac{1}{2}\cos 2A$$

Rectangular/Cartesian Coordinates (x, y, z) $\Leftrightarrow$ Cylindrical Coordinates (ρ, ϕ, z)

$$\rho = \sqrt{x^2 + y^2}\,, \quad \phi = \tan^{-1}\frac{y}{x}\,, \quad z = z \qquad A_\rho = A_x\cos\phi + A_y\sin\phi \qquad A_x = A_\rho\cos\phi - A_\phi\sin\phi \qquad \hat{a}_x\bullet\hat{a}_\rho = \cos\phi \qquad \hat{a}_y\bullet\hat{a}_\phi = \cos\phi$$

$$x = \rho\cos\phi\,, \quad y = \rho\sin\phi\,, \quad z = z \qquad A_\phi = -A_x\sin\phi + A_y\cos\phi \qquad A_y = A_\rho\sin\phi + A_\phi\cos\phi \qquad \hat{a}_x\bullet\hat{a}_\phi = -\sin\phi \qquad \hat{a}_z\bullet\hat{a}_\rho = 0$$

$$\cos\phi = \frac{x}{\sqrt{x^2 + y^2}}\,, \quad \sin\phi = \frac{y}{\sqrt{x^2 + y^2}} \qquad A_z = A_z \qquad A_z = A_z \qquad \hat{a}_y\bullet\hat{a}_\rho = \sin\phi \qquad \hat{a}_z\bullet\hat{a}_\phi = 0$$

Rectangular/Cartesian Coordinates (x, y, z) $\Leftrightarrow$ Spherical Coordinates (r, θ, ϕ)

$$r = \sqrt{x^2 + y^2 + z^2}\,, \quad \phi = \tan^{-1}\frac{y}{x}\,, \quad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z} \qquad x = r\sin\theta\cos\phi\,, \quad y = r\sin\theta\sin\phi\,, \quad z = r\cos\theta$$

$$\cos\phi = \frac{x}{\sqrt{x^2 + y^2}}\,, \qquad \sin\phi = \frac{y}{\sqrt{x^2 + y^2}}\,, \qquad \cos\theta = \frac{z}{\sqrt{x^2 + y^2 + z^2}}\,, \qquad \sin\theta = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}\,,$$

$$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta\,, \qquad A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta\,, \qquad A_\phi = -A_x\sin\phi + A_y\cos\phi$$

$$A_x = A_r\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi\,, \qquad A_y = A_r\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi\,, \qquad A_z = A_r\cos\theta - A_\theta\sin\theta$$

$$\hat{a}_r\bullet\hat{a}_x = \sin\theta\cos\phi\,, \quad \hat{a}_r\bullet\hat{a}_y = \sin\theta\sin\phi\,, \quad \hat{a}_r\bullet\hat{a}_z = \cos\theta\,, \quad \hat{a}_\theta\bullet\hat{a}_x = \cos\theta\cos\phi\,, \quad \hat{a}_\theta\bullet\hat{a}_y = \cos\theta\sin\phi\,, \quad \hat{a}_\theta\bullet\hat{a}_z = -\sin\theta$$

$$\hat{a}_\phi\bullet\hat{a}_x = -\sin\phi\,, \quad \hat{a}_\phi\bullet\hat{a}_y = \cos\phi\,, \quad \hat{a}_\phi\bullet\hat{a}_z = 0$$

Vector Identities

$\int_V \left(\bar{\nabla} \times \bar{A}\right) dV = -\oint_s \bar{A} \times d\bar{s}$, Stoke's Thm. $\int_s \left(\bar{\nabla} \times \bar{A}\right) \bullet d\vec{s} = \oint_c \bar{A} \bullet d\vec{l}$, Divergence Thm. $\int_v \left(\bar{\nabla} \bullet \bar{A}\right) dV = \oint_s \bar{A} \bullet d\vec{s}$,

$\bar{\nabla}(\Phi + \psi) = \bar{\nabla}\Phi + \bar{\nabla}\psi$, $\bar{\nabla}(\Phi\psi) = \Phi\,\bar{\nabla}\psi + \psi\,\bar{\nabla}\Phi$, $\bar{\nabla} \bullet (\psi\,\bar{A}) = \bar{A} \bullet \bar{\nabla}\,\psi + \psi\,\bar{\nabla} \bullet \bar{A}$, $\bar{\nabla} \bullet \bar{\nabla}\Phi = \bar{\nabla}^2\Phi$, $\bar{\nabla} \times \bar{\nabla}\Phi = 0$,

$\bar{\nabla} \bullet \bar{\nabla} \times \bar{A} = 0$, $\bar{\nabla} \bullet (\bar{A} + \bar{B}) = \bar{\nabla} \bullet \bar{A} + \bar{\nabla} \bullet \bar{B}$, $\bar{\nabla} \times (\bar{A} + \bar{B}) = \bar{\nabla} \times \bar{A} + \bar{\nabla} \times \bar{B}$, $\bar{\nabla} \times (\Phi\bar{A}) = \bar{\nabla}\Phi \times \bar{A} + \Phi\bar{\nabla} \times \bar{A}$,

$\bar{\nabla} \bullet (\bar{A} \times \bar{B}) = \bar{B} \bullet \bar{\nabla} \times \bar{A} - \bar{A} \bullet \bar{\nabla} \times \bar{B}$, $\bar{\nabla} \times \bar{\nabla} \times \bar{A} = \bar{\nabla}\left(\bar{\nabla} \bullet \bar{A}\right) - \bar{\nabla}^2\bar{A}$, $\bar{A} \bullet \bar{B} \times \bar{C} = \bar{B} \bullet \bar{C} \times \bar{A} = \bar{C} \bullet \bar{A} \times \bar{B}$,

$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A} \bullet \bar{C}) - \bar{C}(\bar{A} \bullet \bar{B})$

Useful Integrals

$\int \sin ax\,dx = \frac{-1}{a}\cos ax$, $\int \cos ax\,dx = \frac{1}{a}\sin ax$, $\int \sin ax \cos ax\,dx = \frac{1}{2a}\sin^2 ax$, $\int \sin^2 ax\,dx = \frac{1}{2}x - \frac{1}{4a}\sin 2ax$,

$\int \frac{1}{x}dx = \ln|x|$, $\int \cos^2 ax\,dx = \frac{1}{2}x + \frac{1}{4a}\sin 2ax$, $\int x \sin ax\,dx = \frac{1}{a^2}\left[\sin ax - ax\cos ax\right]$,

$\int x \cos ax\,dx = \frac{1}{a^2}\left[\cos ax + ax\sin ax\right]$, $\int \frac{1}{\left(a^2 \pm x^2\right)^{3/2}}dx = \frac{\pm x}{a^2\sqrt{a^2 \pm x^2}}$, $\int \frac{x}{\sqrt{a^2 + x^2}}dx = \sqrt{a^2 + x^2}$,

$\int \frac{x}{\left(a^2 + x^2\right)^{3/2}}dx = \frac{-1}{\sqrt{a^2 \pm x^2}}$, $\int \frac{x^2}{\left(a^2 + x^2\right)^{3/2}}dx = \frac{-x}{\sqrt{a^2 \pm x^2}} + \ln\left(x + \sqrt{a^2 + x^2}\right)$, $\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$,

$\int \frac{x}{a^2 + x^2}dx = \frac{1}{2}\ln\left(a^2 + x^2\right)$, $\int \frac{1}{\sqrt{a^2 + x^2}}dx = \ln\left(x + \sqrt{a^2 + x^2}\right)$, $\int \frac{1}{x\sqrt{a^2 + x^2}}dx = -\frac{1}{a}\ln\left(\frac{a + \sqrt{a^2 + x^2}}{x}\right)$,

$\int_0^{\pi} \frac{a - b\cos x}{a^2 + b^2 - 2ab\cos x}dx = \begin{cases} \frac{\pi}{a} & a > b > 0 \\ 0 & b > a > 0 \end{cases}$, $\int \frac{dx}{x\left(a^2 + x^2\right)^{3/2}} = \frac{1}{a^2\sqrt{a^2 + x^2}} - \frac{1}{a^3}\ln\left(\frac{a + \sqrt{a^2 + x^2}}{a}\right)$