Log-Periodic Dipole Array (LPDA)

Log-Periodic Dipole Array with a coaxial feed (typically 50 $\Omega$ or 75 $\Omega$). [Kraus, Figure 15-13a, p. 708]
LPDA Design Procedure

Based on work of R. E. Carrel.

1. Select or specify design parameters
   a. Desired directivity (gain)
   b. Frequency range ($f_{\text{high}}$ and $f_{\text{low}}$)
   c. Desired input impedance $R_0$ (real)

2. Use graph [Balanis, Figure 11.13, p. 561], which shows contours of constant directivity versus $\sigma$ (relative spacing) and $\tau$ (scale factor), to select $\sigma$ and $\tau$ for the desired directivity.

3. Calculate the apex half angle $\alpha$ using-

$$\alpha = \tan^{-1}\left(\frac{1 - \tau}{4\sigma}\right)$$
Computed Contours of constant directivity versus $\sigma$ and $\tau$ for log periodic dipole arrays. [Balanis, Figure 11.13, p. 561]
LPDA Geometry
4. Find length $l_1$ (Note: start count of elements with longest) of the longest element
- take length in wavelengths from graph if using optimum $\sigma$ and $\tau$;
- else, use $\lambda_{\text{max}}/2$ where $\lambda_{\text{max}} = c/f_{\text{low}}$ is the wavelength at the lowest frequency in the desired frequency range.

Measured length, normalized by $\lambda_{\text{max}}$, of longest dipole in LPDA versus optimum $\sigma$ and $\tau$. 
5. Find length $l_N$ of the **shortest** element
- take length in wavelengths from graph where $\lambda_{\text{min}} = c / f_{\text{high}}$ is the wavelength at the highest frequency in the desired frequency range. This length will be used to know when to truncate (stop) the LPDA. It may or may not be the actual length of the smallest element.

Estimated length, normalized by $\lambda_{\text{min}}$, of shortest dipole in LPDA versus $\sigma$ and $\tau$. 
6. Calculate location $R_1$ of longest element (as measured from the apex)-

$$R_1 = \frac{l_1}{2} \cot(\alpha)$$

7. Calculate the total bandwidth $B_s$, includes additional bandwidth $B_{ar}$ due to active region, using the specified bandwidth $B = \frac{f_{\text{high}}}{f_{\text{low}}}$. 

$$B_{ar} = 1.1 + 7.7 (1-\tau)^2 \cot(\alpha)$$

$$B_s = B_{ar} \cdot B = [1.1 + 7.7 (1-\tau)^2 \cot(\alpha)] B$$

8. Calculate the approximate number $N$ of elements required for design

$$N = 1 + \log_{10}(B_s)/\log_{10}(1/\tau)$$

9. Calculate the approximate distance $L_T$ between the longest and shortest elements.

$$L_T = \frac{l_1}{2} (1 - 1/ B_s) \cot(\alpha) = \frac{\lambda_{\text{max}}}{4} (1 - 1/ B_s) \cot(\alpha)$$
10. Calculate the location $R_2$ (from the apex) and length $l_2$ of the second longest element using the scale factor $\tau$, $R_1$, and $l_1$ -

\[ R_2 = R_1 \tau \quad & \quad l_2 = l_1 \tau \]

11. Recursively calculate the location $R_{n+1}$ and length $l_{n+1}$ of the $n+1$\textsuperscript{th} element(s) using the scale factor $\tau$, $R_n$, and $l_n$ -

\[ R_{n+1} = R_n \tau \quad & \quad l_{n+1} = l_n \tau. \]

Stop when $l_{n+1}$ is less than or equal to $l_N$ (calculated in step 5.).

12. Count actual number of elements and calculate actual length of LPDA (compare to approximate calculations in steps 8. & 9.).

13. Select a length to diameter ratio $K = l/d$ for the elements of the LPDA. This choice is a compromise between mechanical strength for the largest and smallest elements, available tubing sizes, and the selected diameter of the boom.
Available tubing/pipe/rod sizes

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<th>Nominal Diameter (inches)</th>
<th>Outer Diameter* (cm)</th>
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* For brass tubing/pipe/rods, the nominal or outer diameters are the same (wall thickness negligible). For copper pipes, the wall thickness is substantial and should be measured as it varies between manufacturers.
14. Calculate the diameter $d_n$ for each element. Then, select the closest available tube/pipe/rod diameter to the calculated value.

$$d_n = l_n / K$$

15. Calculate the actual length to diameter ratio $K_n$ for each element and the average length to diameter ratio $K_{\text{ave}}$ after quantization. Check for unusually large deviations from desired $K$ (may want to go back to step 13. and select another value of $K$).

16. Calculate the approximate average characteristic impedance of the active region elements-

$$Z_a = 60 \ln(2 X K_{\text{ave}} / \pi)$$

where $X = 8 \tau \sigma / (1 + \tau)$. 
17. Find the characteristic impedance of the unloaded transmission line 
\( Z_0 \) for the desired input impedance \( R_0 \)-

\[
Z_0 = \frac{R_0^2}{4Z_a X} + R_0 \sqrt{\left( \frac{R_0}{4Z_a X} \right)^2 + 1}
\]

18. Calculate the center-to-center spacing \( S \) of the booms using the 
unloaded, cylindrical, twin-lead transmission line formula-

\[
S = D \cosh\left(\frac{Z_0}{120}\right)
\]

where \( D \) is the diameter of the booms (assumed to be identical). The 
air gap \( \Delta_{\text{gap}} \) between the inner surfaces of the booms is \( \Delta_{\text{gap}} = S - D \).